1. The distances travelled to work, D km, by the employees at a large company are normally distributed with $D \sim \mathrm{N}(30, 8^2)$.

(a) Find the probability that a randomly selected employee has a journey to work of more than 20 km.

(3)

(b) Find the upper quartile, Q_3, of D.

(3)

(c) Write down the lower quartile, Q_1, of D.

(1)

An outlier is defined as any value of D such that $D < h$ or $D > k$ where

$$h = Q_1 - 1.5 \times (Q_3 - Q_1) \quad \text{and} \quad k = Q_3 + 1.5 \times (Q_3 - Q_1)$$

(d) Find the value of h and the value of k.

(2)

An employee is selected at random.

(e) Find the probability that the distance travelled to work by this employee is an outlier.

(3)

(Total 12 marks)

1.	(a)	$P(D > 20) = P\left(Z > \frac{20-30}{8}\right)$		M1	
		$= P(Z > -1.25)$		A1	
		$=$ **<u>0.8944</u>**	**<u>awrt 0.894</u>**	A1	3

<u>Note</u>

M1 for an attempt to standardise 20 or 40 using 30 and 8.
1st A1 for $z = \pm 1.25$
2nd A1 for awrt 0.894

(b)	$P(D < Q_3) = 0.75$ so $\frac{Q_3 - 30}{8} = 0.67$		M1 B1	
	$Q_3 =$ **awrt <u>35.4</u>**		A1	3

<u>Note</u>

M1 for $\frac{Q_3 - 30}{8}$ = to a z value

M0for 0.7734 on RHS.
B1 for (z value) between 0.67 ~ 0.675 seen.
M1B0A1 for use of $z = 0.68$ in correct expression with awrt 35.4

(c)	35.4 – 30= 5.4 so $Q_1 = 30 - 5.4$ = **awrt <u>24.6</u>**		B1ft	1

<u>Note</u>

Follow through using their of quartile values.

(d)	$Q_3 - Q_1 = 10.8$ so $1.5(Q_3 - Q_1) = 16.2$ so $Q_1 - 16.2 = h$			
	or $Q_3 + 16.2 = k$		M1	
	h=**<u>8.4 to 8.6</u>** and k= **<u>51.4 to 51.6</u>**	both	A1	2

<u>Note</u>

M1 for an attempt to calculate 1.5(IQR) and attempt to add or subtract using one of the formulae given in the question – follow through their quartiles

(e)	$2P(D > 51.6) = 2P(Z > 2.7)$		M1	
	$= 2[1 - 0.9965] =$ awrt **<u>0.007</u>**		M1 A1	3

<u>Note</u>

1st M1 for attempting $2P(D >$ their $k)$ or ($P(D >$their $k) + P(D <$ their $h)$)
2nd M1 for standardising their h or k (may have missed the 2) so allow for standardising $P(D > 51.6)$ or $P(D < 8.4)$
Require boths Ms to award A mark.

[12]

2. The heights of a population of women are normally distributed with mean μ cm and standard deviation σ cm. It is known that 30% of the women are taller than 172 cm and 5% are shorter than 154 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information.

(3)

(b) Show that $\mu = 154 + 1.6449\sigma$.

(3)

(c) Obtain a second equation and hence find the value of μ and the value of σ.

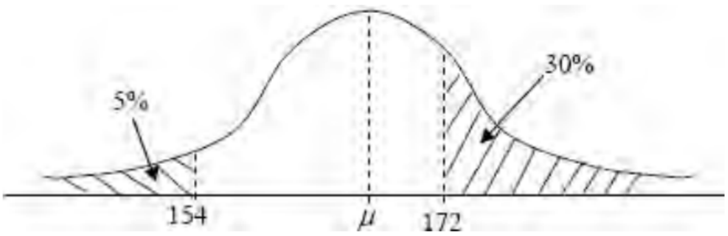
(4)

A woman is chosen at random from the population.

(d) Find the probability that she is taller than 160 cm.

(3)
(Total 13 marks)

2. (a)



bell shaped, must have inflexions	B1	
154,172 on axis	B1	
5% and 30%	B1	3

Note

2nd B1 for 154 and 172 marked but 154 must be $< \mu$ and $172 > \mu$. But μ need not be marked.

Allow for $\frac{154-\mu}{\sigma}$ and $\frac{172-\mu}{\sigma}$ marked on appropriate sides of the peak.

3rd B1 the 5% and 30% should be clearly indicated in the correct regions i.e. LH tail and RH tails.

(b)

$P(X<154) = 0.05$	M1	
$\frac{154-\mu}{\sigma} = -1.6449$ or $\frac{\mu-154}{\sigma} = 1.6449$	B1	
$\mu = 154 + 1.6449\sigma$ * * given * *	A1 cso	3

Note

M1 for $\pm\frac{(154-\mu)}{\sigma} = z$ value (z must be recognizable e.g. 1.64, 1.65, 1.96 but NOT 0.5199 etc)

B1 for $\pm$ 1.6449 seen in a line before the final answer.

A1cso for no incorrect statements (in μ, σ) equating a z value and a probability or incorrect signs e.g. $\frac{154-\mu}{\sigma}=0.05$ or $\frac{154-\mu}{\sigma}=1.6449$ or $P(Z<\frac{\mu-154}{\sigma})=1.6449$

(c)

$172 - \mu = 0.5244\sigma$ or $\frac{172-\mu}{\sigma} = 0.5244$ (allow z = 0.52 or better here but must be in an equation)	B1	
Solving gives $\sigma = 8.2976075$ (**awrt 8.30**) and $\mu = 167.64873$ (**awrt 168**)	M1 A1 A1	4

Note

B1 for a correct 2nd equation (NB 172 $-\mu = 0.525\sigma$ is B0, since z is incorrect)

M1 for solving their two linear equations leading to $\mu = ...$ or $\sigma = ...$

1st A1 for σ = awrt 8.30, 2nd A1 for μ = awrt 168 [NB the 168 can come from false working.

These A marks require use of correct equation from (b), and a z value for "0.5244" in (c)]

NB use of z = 0.52 will typically get σ =8.31 and μ = 167.67… and score B1M1A0A1

No working and both correct scores 4/4, only one correct scores 0/4

Provided the M1 is scored the A1s can be scored even with B0 (e.g. for z =0.525)

(d) P(Taller than 160cm) = $P\left(Z > \frac{160-\mu}{\sigma}\right)$		M1	
$= P(Z < 0.9217994)$		B1	
$= 0.8212$	**awrt 0.82**	A1	3

Note

M1 for attempt to standardise with 160, their μ and their σ (> 0). Even allow with symbols μ and σ.

B1 for z = awrt $\pm$ 0.92

No working and a correct answer can score 3/3 provided σ and μ are correct to 2sf.

[13]

3. The lifetimes of bulbs used in a lamp are normally distributed.
A company X sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

(a) Find the probability of a bulb, from company X, having a lifetime of less than 830 hours.
(3)

(b) In a box of 500 bulbs, from company X, find the expected number having a lifetime of less than 830 hours.
(2)

A rival company Y sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less than 818 hours.

(c) Find the standard deviation of the lifetimes of bulbs from company Y.
(4)

Both companies sell the bulbs for the same price.

(d) State which company you would recommend. Give reasons for your answer.
(2)
(Total 11 marks)

3.	(a)	Let the random variable X be the lifetime in hours of bulb			
		$P(X < 830) = P(Z < \frac{\pm(830-850)}{50})$	Standardising with 850 and 50	M1	
		$= P(Z < -0.4)$			
		$= 1 - P(Z < 0.4)$	Using 1-(probability >0.5)	M1	
		$= 1 - 0.6554$			
		$= 0.3446$ or 0.344578 by calculator	awrt 0.345	A1	3
		Note			
		If 1-z used e.g. $1 - 0.4 = 0.6$ then award second M0			
	(b)	0.3446×500	Their (a) × 500	M1	
		$= 172.3$	Accept 172.3 or 172 or 173	A1	2
	(c)	Standardise with 860 and σ and equate to z value $\frac{\pm(818-860)}{\sigma} = z$ value		M1	
		$\frac{818-860}{\sigma} = -0.84(16)$ or $\frac{860-818}{\sigma} = 0.84(16)$ or $\frac{902-860}{\sigma} = 0.84(16)$ or equiv.		A1	
		$\sigma = 49.9$	±0.8416(2)	B1	
			50 or awrt 49.9	A1	4
		Note			
		M1 can be implied by correct line 2			
		A1 for completely correct statement or equivalent.			
		Award B1 if 0.8416(2) seen			
		Do not award final A1 if any errors in solution e.g. negative sign lost.			
	(d)	Company Y as the ***mean*** is greater for Y.	both	B1	
		They have (approximately) the same ***standard deviation*** or ***sd***		B1	2
		Note			
		Must use statistical terms as underlined.			

[11]

10. The heights of a group of athletes are modelled by a normal distribution with mean 180 cm and standard deviation 5.2 cm. The weights of this group of athletes are modelled by a normal distribution with mean 85 kg and standard deviation 7.1 kg.

Find the probability that a randomly chosen athlete,

(a) is taller than 188 cm,

(3)

(b) weighs less than 97 kg.

(2)

(c) Assuming that for these athletes height and weight are independent, find the probability that a randomly chosen athlete is taller than 188 cm and weighs more than 97 kg.

(3)

(d) Comment on the assumption that height and weight are independent.

(1)

(Total 9 marks)

10.	(a)	Let H be rv height of athletes, so $H \sim N(180, 5.2^2)$		
		$P(H > 188) = P(Z > \frac{188-180}{5.2}) = P(Z > 1.54) = 0.0618$		
		± stand √, sq, awrt 0.062	M1 A1 A1	3
	(b)	Let W be rv weight of athletes, so $W \sim N(85, 7.1^2)$	M1 A1	2
		$P(W < 97) = P(Z < 1.69) = 0.9545$ standardise, awrt 0.9545	M1 A1ft	
	(c)	$P(H > 188 \;\&\; W < 97) = 0.0618(1–0.9545)$ allow (a)×(b) for M		
		$= 0.00281$ awrt 0.0028	A1	3
	(d)	Evidence suggests height and weight are positively correlated / linked		
		Assumption of independence is not sensible	B1	1

[9]

19. A drinks machine dispenses coffee into cups. A sign on the machine indicates that each cup contains 50 ml of coffee. The machine actually dispenses a mean amount of 55 ml per cup and 10% of the cups contain less than the amount stated on the sign. Assuming that the amount of coffee dispensed into each cup is normally distributed find

(a) the standard deviation of the amount of coffee dispensed per cup in ml,

(4)

(b) the percentage of cups that contain more than 61 ml.

(3)

Following complaints, the owners of the machine make adjustments. Only 2.5% of cups now contain less than 50 ml. The standard deviation of the amount dispensed is reduced to 3 ml.

Assuming that the amount of coffee dispensed is still normally distributed,

(c) find the new mean amount of coffee per cup.

(4)
(Total 11 marks)

19. Let X represent amount dispersed into cups
$\therefore X \sim N(55, \sigma)$

(a) $P(X < 50) = 0.10 \Rightarrow \dfrac{50-55}{\sigma} = -1.2816$ M1 B1

$\sigma = 3.90137$ M1 A1 4

(b) $P(X > 61) = P(Z > \dfrac{61-55}{3.90137...})$ M1

$= P(Z > 1.54)$ A1

$= 1 - 0.90382 = 0.0618$; 6.18% A1 3

(c) Let Y represent new amount dispensed.
$\therefore Y \sim N(\mu, 3)$

$P(Y < 50) = 0.025 \Rightarrow \dfrac{50-\mu}{3} = -1.96$ M1 B1

$\mu = 55.88$ M1 A1 4

[11]

20. The weight of coffee in glass jars labelled 100 g is normally distributed with mean 101.80 g and standard deviation 0.72 g. The weight of an empty glass jar is normally distributed with mean 260.00 g and standard deviation 5.45 g. The weight of a glass jar is independent of the weight of the coffee it contains.

Find the probability that a randomly selected jar weighs less than 266 g and contains less than 100 g of coffee. Give your answer to 2 significant figures.

(Total 8 marks)

20. Let J represent the weight of a Jar $\therefore J \sim N(260.00, 5.45^2)$

$\therefore P(J < 266) = P\left(Z < \frac{266-260}{5.45}\right)$ M1 A1

$= P(Z < 1.10)$

$= 0.8643$ A1

(NB: calculator gives 0.86453: accept 0.864 – 0.865)

Let C represent weight of coffee in a Jar $\therefore C \sim N(101.8, 0.72^2)$

$\therefore P(C < 100) = P\left(Z < \frac{100-101.8}{0.72}\right)$ M1 A1

$= P(Z < -2.50)$ A1

$= 0.0062$

$\therefore P(J < 266 \text{ \& } C < 100) = 0.8643 \times 0.0062$ M1

$= 0.0054$ A1 8

[8]