

10

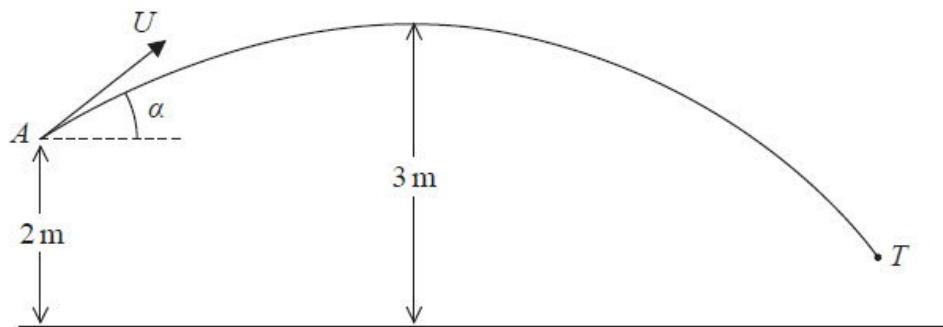


Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A , the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground. The target is modelled as being the point T , as shown in Figure 4.

The ball is modelled as a particle moving freely under gravity.

Using the model,

$$(a) \text{ show that } U^2 = \frac{2g}{\sin^2 \alpha}. \quad (2)$$

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

$$(b) \text{ Find the size of the angle } \alpha \quad (9)$$

$$(c) \text{ State one limitation of the model that could affect your answer to part (b).} \quad (1)$$

$$(d) \text{ Find the time taken for the ball to travel from } A \text{ to } T. \quad (3)$$

Question	Scheme	Marks	AOs
10(a)	Using the model and vertical motion: $0^2 = (U \sin \alpha)^2 - 2g \leftrightarrow (3-2)$	M1	3.3
	$U^2 = \frac{2g}{\sin^2 \alpha} *$ GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut \cos \alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-\frac{5}{4} = Ut \sin \alpha - \frac{1}{2}gt^2$	A1	1.1b
	sub for t : $-\frac{5}{4} = U \sin \alpha \left(\frac{20}{U \cos \alpha} \right) - \frac{1}{2}g \left(\frac{20}{U \cos \alpha} \right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\frac{5}{4} = 20 \tan \alpha - 100 \tan^2 \alpha$	A1(I)	1.1b
	$(4 \tan \alpha - 1)(100 \tan \alpha + 5) = 0$	M1(III)	1.1b
	$\tan \alpha = \frac{1}{4} \Rightarrow \alpha = 14^\circ$ or better	A1(II)	2.2a
		(9)	
	N.B. For the last 5 marks, they may set up a quadratic in t , by substituting for $U \sin \alpha$ first, then solve the quadratic to find the value of t , then use $20 = Ut \cos \alpha$ to find α . The marks are the same but earned in a different order. Enter on ePen in the corresponding M and A boxes above, as indicated below.		
	Sub for $U \sin \alpha$ to give equation in t only	M1(II)	
	$-\frac{5}{4} = \sqrt{2g}t - \frac{1}{2}gt^2$	A1(I)	
	Solve for t	M1(III)	
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13 and use $20 = Ut \cos \alpha$	M1(I)	
	$\alpha = 14^\circ$ or better	A1(II)	
(b)	ALTERNATIVE		

1 Fig. 7 shows the trajectory of an object which is projected from a point O on horizontal ground. Its initial velocity is 40 ms^{-1} at an angle of α to the horizontal.

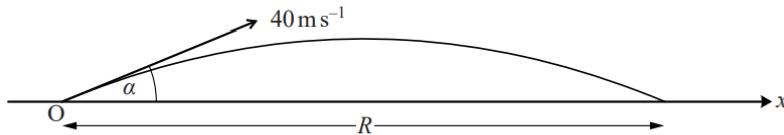


Fig. 7

(i) Show that, according to the standard projectile model in which air resistance is neglected, the flight time, T s, and the range, R m, are given by

$$T = \frac{80 \sin \alpha}{g} \text{ and } R = \frac{3200 \sin \alpha \cos \alpha}{g}. \quad [6]$$

A company is designing a new type of ball and wants to model its flight.

(ii) Initially the company uses the standard projectile model.

Use this model to show that when $\alpha = 30^\circ$ and the initial speed is 40 ms^{-1} , T is approximately 4.08 and R is approximately 141.4.

Find the values of T and R when $\alpha = 45^\circ$. [3]

The company tests the ball using a machine that projects it from ground level across horizontal ground. The speed of projection is set at 40 ms^{-1} .

When the angle of projection is set at 30° , the range is found to be 125 m.

(iii) Comment briefly on the accuracy of the standard projectile model in this situation. [1]

The company refines the model by assuming that the ball has a constant deceleration of 2 ms^{-2} in the horizontal direction.

In this new model, the resistance to the vertical motion is still neglected and so the flight time is still 4.08 s when the angle of projection is 30° .

(iv) Using the new model, with $\alpha = 30^\circ$, show that the horizontal displacement from the point of projection, x m at time t s, is given by

$$x = 40t \cos 30^\circ - t^2.$$

Find the range and hence show that this new model is reasonably accurate in this case. [4]

The company then sets the angle of projection to 45° while retaining a projection speed of 40 ms^{-1} . With this setting the range of the ball is found to be 135 m.

(v) Investigate whether the new model is also accurate for this angle of projection. [3]

(vi) Make one suggestion as to how the model could be further refined. [1]

2

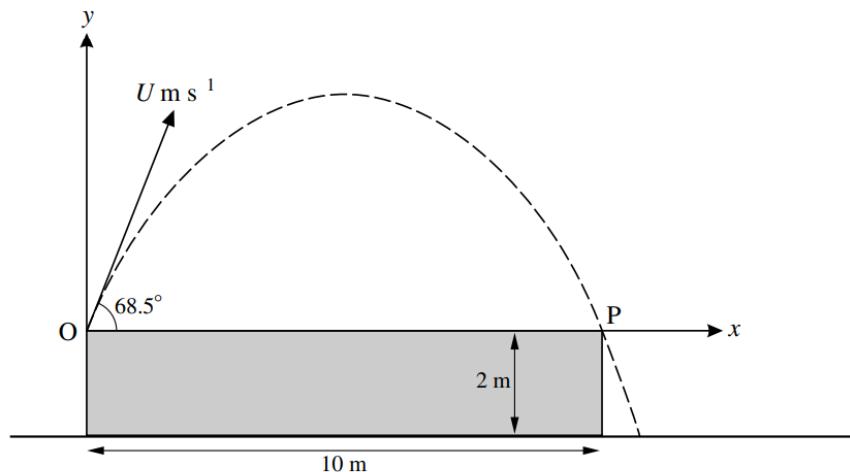


Fig. 7

Fig. 7 shows a platform 10 m long and 2 m high standing on horizontal ground. A small ball projected from the surface of the platform at one end, O, just misses the other end, P. The ball is projected at 68.5° to the horizontal with a speed of $U \text{ m s}^{-1}$. Air resistance may be neglected.

At time t seconds after projection, the horizontal and vertical displacements of the ball from O are x m and y m.

(i) Obtain expressions, in terms of U and t , for

(A) x ,

(B) y .

[3]

(ii) The ball takes T s to travel from O to P.

Show that $T = \frac{U \sin 68.5^\circ}{4.9}$ and write down a second equation connecting U and T .

[4]

(iii) Hence show that $U = 12.0$ (correct to three significant figures).

[3]

(iv) Calculate the horizontal distance of the ball from the platform when the ball lands on the ground.

[5]

(v) Use the expressions you found in part (i) to show that the cartesian equation of the trajectory of the ball in terms of U is

$$y = x \tan 68.5^\circ - \frac{4.9x^2}{U^2(\cos 68.5^\circ)^2}.$$

Use this equation to show again that $U = 12.0$ (correct to three significant figures).

[4]

2		mark	notes
(i) (A)	$x = Ut \cos 68.5^\circ$	B1 1	
(i) (B)	$y = Ut \sin 68.5^\circ - 4.9t^2$	M1 A1 2	Allow 'u' = U . Allow $s \leftrightarrow c$. Allow g as g , $\pm 9.8, \pm 9.81, \pm 10$. Allow +2. Accept not 'shown'. Do not allow +2. Allow e.g. $+0.5 \times (-9.8) \times t^2$ instead of $-4.9t^2$. Accept g not evaluated
(ii)	<p>either At D, $y = 0$ so $U \sin 68.5^\circ T - 4.9 \times T^2 = 0$ $\Rightarrow T(U \sin 68.5^\circ - 4.9T) = 0$</p> <p>so $T = 0$ (at C) or $T = \frac{U \sin 68.5^\circ}{4.9}$ (at D)</p> <p>or</p> <p>Use (i)(A) and put $x = 10$ with $t = T$ to get $UT \cos 68.5^\circ = 10$</p>	M1 M1 E1 M1 M1 E1 B1 4	Equating correct y to 0 or their y to correct value. Attempting to factorise (or solve). Allow $\div T$ without comment. Properly shown. Accept no ref to $T = 0$. Accept $T = 0$ given as well without comment. Find time to top Double time to the top Properly shown. Accept no ref to $T = 0$. Accept $T = 0$ given as well without comment. Find time to top Double time to the top
(iii)	<p>Eliminating T from the results in (ii) gives</p> $U \cos 68.5^\circ \times \frac{U \sin 68.5^\circ}{4.9} = 10$ <p>so $U = 11.98729\dots$ so 12.0 (3 s. f.)</p>	M1 M1 E1 3	Substituting, using correct expressions or their expressions from (ii). Attempt to solve for U^2 or U . Some evidence seen. e.g. $142.8025\dots < U^2 < 145.2025\dots$ with clear statement, or 11.9... seen with clear statement or 11.98... seen. Accept 11.98... seen for full marks.
(iv)	continued		
(iv)	<p>Require $U t \sin 68.5^\circ - 4.9t^2 = -2$ Solving $4.9t^2 - Ut \sin 68.5^\circ - 2 = 0$</p> <p>$t = -0.1670594541\dots, 2.4431591\dots$ (Using 12; $-0.1669052502\dots, 2.445478886\dots$)</p> <p>Require $U \cos 68.5^\circ \times 2.44\dots - 10$ $= 0.7336\dots$ so 0.734 m (3 s. f.) (Using 12 consistently, 0.7552... so 0.755 (3 s. f.))</p>	M1 M1 A1 M1 A1 5	Equating correct y to -2 or their y to correct value. Allow use of U , 11.987... or 12. Allow implicit ' $= 0$ ' Dep on 1 st M1. Attempt to solve a 3 term quadratic to find at least the +ve root. Allow if two correct roots seen WW. Accept only + ve root given Alternative method of e.g. finding time to highest point and then time to the ground. M1 all times attempted, at least one by a sound method. M1 both methods sound and complete. A1. Dep on first M1. Allow their expression for x . Allow ' -10 ' omitted. cao. Accept $0.73 \leq x \leq 0.76$
(v)	<p>Eliminate t from (i) (B)</p> <p>using $t = \frac{x}{U \cos 68.5^\circ}$ from (i)(A)</p> <p>so $y = x \tan 68.5^\circ - \frac{4.9x^2}{U^2 (\cos 68.5^\circ)^2}$</p> <p>We require $y = 0$ when $x = 10$</p> <p>so $U = 11.98729\dots$ so 12.0 (3 s. f.)</p>	M1 E1 M1 E1 4 19	May be implied. FT their (i). Clearly shown. Must see attempt to solve. Or use $x = 10.73\dots$ when $y = -2$. Must see evidence of fresh calculation or statement that they have now got the same expression for evaluation.

3

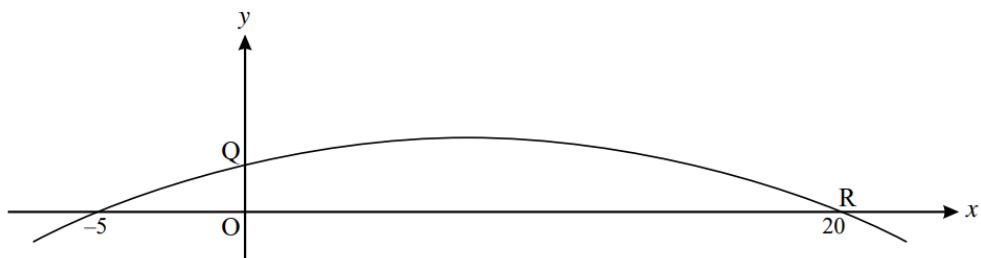


Fig. 7

Fig. 7 shows the graph of $y = \frac{1}{100}(100 + 15x - x^2)$.

For $0 \leq x \leq 20$, this graph shows the trajectory of a small stone projected from the point Q where y m is the height of the stone above horizontal ground and x m is the horizontal displacement of the stone from O. The stone hits the ground at the point R.

- (i) Write down the height of Q above the ground. [1]
- (ii) Find the horizontal distance from O of the highest point of the trajectory and show that this point is 1.5625 m above the ground. [5]
- (iii) Show that the time taken for the stone to fall from its highest point to the ground is 0.565 seconds, correct to 3 significant figures. [3]
- (iv) Show that the horizontal component of the velocity of the stone is 22.1 m s^{-1} , correct to 3 significant figures. Deduce the time of flight from Q to R. [5]
- (v) Calculate the speed at which the stone hits the ground. [4]

3 (i)	$y(0) = 1$	B1		1
(ii)	<p>Either $\frac{1}{2}(20+5) - 5 = 7.5$</p> <p>or</p> $y(7.5) = \frac{1}{100}(100 + 15 \times 7.5 - 7.5^2)$ $= \frac{25}{16}(1.5625) \text{ so } 1.5625 \text{ m}$	M1 A1 A1 M1 A1 A1 M1 E1	Use of symmetry e.g. use of $\frac{1}{2}(20+5)$ 12.5 o.e. seen 7.5 cao Att pt at y' and to solve $y' = 0$ $k(15 - 2x)$ where $k = 1$ or $\frac{1}{100}$ 7.5 cao, seen as final answer FT their 7.5 A [SC2 only showing 1.5625 leads to $x = 7.5$]	5

(iii)	$4.9t^2 = \frac{25}{16}(1.5625)$ $t^2 = 0.31887\dots$ so $t = \pm 0.56469\dots$ Hence 0.565 s (3 s. f.)	M1 A1 E1	Use of $s = ut + 0.5at^2$ with $u = 0$. Condone use of $\pm 10, \pm 9.8, \pm 9.81$. If sequence of suvat used, complete method required. In any method only error accepted is sign error AG. Condone no reference to -ve value. www. 0.565 must be justified as answer to 3 s. f.	3
(iv)	$\dot{x} = \frac{12.5}{0.56469\dots} = 22.1359\dots$ so 22.1 m s^{-1} (3 s. f.) <p>Either Time is $\frac{20}{12.5} \times 0.56469\dots$ s so 0.904 s (3 s. f.) or Time is $\frac{20}{22.1359\dots}$ s $= 0.903507\dots$ so 0.904 s (3 s. f.) or $(\text{iii}) + \frac{7.5}{\text{their } \dot{x}}$ so 0.904 s (3 s. f.)</p>	M1 B1 E1 M1 A1 M1 A1 M1 A1	or $25 / (2 \times 0.56469\dots)$ Use of 12.5 or equivalent 22.1 must be justified as answer to 3 s. f. Don't penalise if penalty already given in (iii). cao Accept 0.91 (2 s. f.) cao Accept 0.91 (2 s. f.) cao Accept 0.91 (2 s. f.)	5
(v)	$v = \sqrt{\dot{x}^2 + \dot{y}^2}$ $\dot{y}^2 = 0^2 + 2 \times 9.8 \times \frac{25}{16}$ or $\dot{y} = 0 + 9.8 \times 0.5646\dots$ $= \frac{245}{8}(30.625) \quad \dot{y} = \pm 5.539\dots$ so $v = \sqrt{490 + 30.625} = 22.8172\dots \text{ m s}^{-1}$ so 22.8 m s^{-1} (3 s. f.)	M1 M1 A1 A1	Must have attempts at both components Or equiv. $u = 0$. Condone use of $\pm 10, \pm 9.8, \pm 9.81$. Accept wrong s (or t in alternative method) Or equivalent. May be implied. Could come from (iii) $v^2 = u^2 + 2as$ used there. Award marks again. cao. www	4

4 Sandy is throwing a stone at a plum tree. The stone is thrown from a point O at a speed of 35 m s^{-1} at an angle of α to the horizontal, where $\cos \alpha = 0.96$. You are given that, t seconds after being thrown, the stone is $(9.8t - 4.9t^2)$ m higher than O.

When descending, the stone hits a plum which is 3.675 m higher than O. Air resistance should be neglected.

Calculate the horizontal distance of the plum from O.

[6]

5 Small stones A and B are initially in the positions shown in Fig. 6 with B a height H m directly above A.

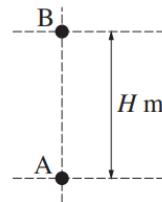


Fig. 6

At the instant when B is released from rest, A is projected vertically upwards with a speed of 29.4 m s^{-1} . Air resistance may be neglected.

The stones collide T seconds after they begin to move. At this instant they have the same speed, $V \text{ m s}^{-1}$, and A is still rising.

By considering when the speed of A upwards is the same as the speed of B downwards, or otherwise, show that $T = 1.5$ and find the values of V and H .

[7]

4		Mark	Comment	Sub
	<p>either We need $3.675 = 9.8t - 4.9t^2$</p> <p>Solving $4t^2 - 8t + 3 = 0$</p> <p>gives $t = 0.5$ or $t = 1.5$</p> <p>or</p> <p>Time to greatest height $0 = 35 \times 0.28 - 9.8t$ so $t = 1$</p> <p>Time to drop is 0.5 total is 1.5 s</p> <p>then Horiz distance is $35 \times 0.96t$ So distance is $35 \times 0.96 \times 1.5 = 50.4$ m</p>	*M1 M1* A1 F1 M1 A1 A1 A1 B1 F1	<p>Equating given expression or their attempt at y to ± 3.675. If they attempt y, allow sign errors, $g = 9.81$ etc. and $u = 35$.</p> <p>Dependent. Any method of solution of a 3 term quadratic.</p> <p>cao. Accept only the larger root given</p> <p>Both roots shown and larger chosen provided both +ve. Dependent on 1st M1.</p> <p>[Award M1 M1 A1 for 1.5 seen WW]</p> <p>Complete method for total time from motion in separate parts. Allow sign errors, $g = 9.81$ etc. Allow $u = 35$ initially only.</p> <p>Time for 1st part</p> <p>Time for 2nd part</p> <p>cao</p> <p>Use of $x = u \cos \alpha t$. May be implied.</p> <p>FT their quoted t provided it is positive.</p>	6

5		Mark	Comment	Sub
	<p>Method 1</p> <p>$\uparrow v_A = 29.4 - 9.8T \quad \downarrow v_B = 9.8T$</p> <p>For same speed $29.4 - 9.8T = 9.8T$</p> <p>so $T = 1.5$ and $V = 14.7$</p> <p>$H = 29.4 \times 1.5 - 0.5 \times 9.8 \times 1.5^2$ $+ 0. \times 9.8 \times 1.5^2$ $= 44.1$</p> <p>Method 2</p> <p>$V^2 = 29.4^2 - 2 \times 9.8 \times x = 2 \times 9.8 \times (H - x)$</p> <p>$29.4^2 = 19.6H$ so $H = 44.1$</p> <p>Relative velocity is 29.4 so</p> <p>$T = \frac{44.1}{29.4}$</p> <p>Using $v = u + at$ $V = 0 + 9.8 \times 1.5 = 14.7$</p>	M1 A1 M1 E1 F1 M1 A1 M1 B1 A1 M1 M1 E1 M1 F1	<p>Either attempted. Allow sign errors and $g = 9.81$ etc</p> <p>Both correct</p> <p>Attempt to equate. Accept sign errors and $T = 1.5$ substituted in both.</p> <p>If 2 subs there must be a statement about equality</p> <p>FT T or V, whichever is found second</p> <p>Sum of the distance travelled by each attempted</p> <p>cao</p> <p>Attempts at V^2 for each particle equated. Allow sign errors, 9.81 etc</p> <p>Allow h_1, h_2 without $h_1 = H - h_2$</p> <p>Both correct. Require $h_1 = H - h_2$ but not an equation.</p> <p>cao</p> <p>Any method that leads to T or V</p> <p>Any method leading to the other variable</p> <p>Other approaches possible. If 'clever' ways seen, reward according to weighting above.</p>	7

1 A small firework is fired from a point O at ground level over horizontal ground. The highest point reached by the firework is a horizontal distance of 60 m from O and a vertical distance of 40 m from O, as shown in Fig. 7. Air resistance is negligible.

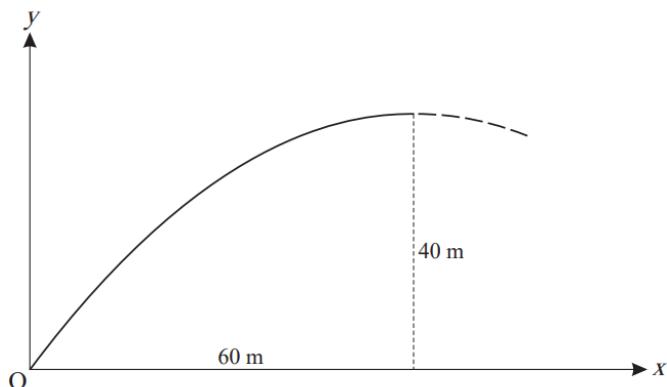


Fig. 7

The initial horizontal component of the velocity of the firework is 21 m s^{-1} .

(i) Calculate the time for the firework to reach its highest point and show that the initial vertical component of its velocity is 28 m s^{-1} . [4]

(ii) Show that the firework is $(28t - 4.9t^2)$ m above the ground t seconds after its projection. [1]

When the firework is at its highest point it explodes into several parts. Two of the parts initially continue to travel horizontally in the original direction, one with the original horizontal speed of 21 m s^{-1} and the other with a quarter of this speed.

(iii) State why the two parts are always at the same height as one another above the ground and hence find an expression in terms of t for the distance between the parts t seconds after the explosion. [3]

(iv) Find the distance between these parts of the firework

(A) when they reach the ground, [2]

(B) when they are 10 m above the ground. [5]

(v) Show that the cartesian equation of the trajectory of the firework before it explodes is $y = \frac{1}{90}(120x - x^2)$, referred to the coordinate axes shown in Fig. 7. [4]

1		Mark	Comment	
(i)	<p>Hor $21t = 60$ so $\frac{20}{7}$ s (2.8571...)</p> <p>either $0 = u - 9.8 \times \frac{20}{7}$ or $-u = u - 9.8 \times \left(\frac{40}{7}\right)$ or $40 = u \times \frac{20}{7} - 4.9 \left(\frac{20}{7}\right)^2$ so $u = 28$ so 28 m s^{-1}</p>	M1 A1 M1 E1	Use of horizontal components and $a = 0$ or $s = vt - 0.5at^2$ with $v = 0$. Any form acceptable. Allow M1 A1 for answer seen WW. [If $s = ut + 0.5at^2$ and $u = 0$ used without justification award M1 A0] [If $u = 28$ assumed to find time then award SC1] Use of $v = u + at$ (or $v^2 = u^2 + 2as$) with $v = 0$. or Use of $v = u + at$ with $v = -u$ and appropriate t . or Use of $s = ut + 0.5at^2$ with $s = 40$ and appropriate t Condone sign errors and, where appropriate, $u \leftrightarrow v$. Accept signs not clear but not errors. Enough working must be given for 28 to be properly shown. [NB $u = 28$ may be found first and used to find time]	4
(ii)	$y = 28t - 0.5 \times 9.8t^2$	E1	<i>Clear & convincing</i> use of $g = -9.8$ in $s = ut + 0.5at^2$ or $s = vt - 0.5at^2$ NB: AG	1
(iii)	<p>Start from same height with same (zero) vertical speed at same time, same acceleration</p> <p>Distance apart is $0.75 \times 21t = 15.75t$</p>	E1 M1 A1	For two of these reasons $0.75 \times 21t$ seen or $21t$ and $5.25t$ both seen with intention to subtract. Need simplification - LHS alone insufficient. CWO.	3
(iv) (A)	<p>either Time is $\frac{20}{7}$ s by symmetry so $15.75 \times \frac{20}{7} = 45$ so 45 m</p> <p>or Hit ground at same time. By symmetry one travels 60 m so the other travels 15 m in this time ($\frac{1}{4}$ speed) so 45 m.</p>	B1 B1 B1 B1	Symmetr or $uvast$ FT their (iii) with $t = \frac{20}{7}$ [SC1 if 90 m seen]	2
(B)	see next page			

1	continued			
(B)	<p>either Time to fall is $40 - 10 = 0.5 \times 9.8 \times t^2$</p> <p>$t = 2.47435\dots$ need $15.75 \times 2.47435\dots = 38.971\dots$ so 39.0 (3sf)</p> <p>or Need time so $10 = 28t - 4.9t^2$ $4.9t^2 - 28t + 10 = 0$ so $t = \frac{28 \pm \sqrt{28^2 - 4 \times 4.9 \times 10}}{9.8}$ so 0.382784\dots or 5.33150\dots</p> <p>Time required is $5.33150\dots - \frac{20}{7} = 2.47435\dots$ need $15.75 \times 2.47435\dots = 38.971\dots$ so 39.0 (3sf)</p>	<p>M1 [SC1 if either and or methods mixed to give $\pm 30 = 28t - 4.9t^2$ or $\pm 10 = 4.9t^2$]</p> <p>A1 Considering time from explosion with $u = 0$. Condone sign errors. LHS. Allow ± 30</p> <p>A1 All correct cao</p> <p>F1 FT their (iii) only.</p> <p>M1 Equating $28t - 4.9t^2 = \pm 10$ Dep. Attempt to solve quadratic by a method that could give two roots.</p> <p>A1 Larger root correct to at least 2 s. f. Both method marks may be implied from two correct roots alone (to at least 1 s. f.). [SC1 for either root seen WW]</p> <p>M1</p> <p>F1 FT their (iii) only.</p>		5
(v)	<p>Horiz ($x =$) $21t$ Elim t between $x = 21t$ and $y = 28t - 4.9t^2$</p> <p>so $y = 28\left(\frac{x}{21}\right) - 4.9\left(\frac{x}{21}\right)^2$</p> <p>so $y = \frac{4x}{3} - \frac{0.1x^2}{9} = \frac{1}{90}(120x - x^2)$</p>	<p>B1</p> <p>M1 Intention must be clear, with some attempt made.</p> <p>A1 t completely and correctly eliminated from their expression for x and correct y. Only accept wrong notation if subsequently explicitly given correct value e.g. $\frac{x^2}{21}$ seen as $\frac{x^2}{441}$.</p> <p>E1 Some simplification must be shown. [SC2 for 3 points shown to be on the curve. Award more only if it is made clear that (a) trajectory is a parabola (b) 3 points define a parabola]</p>		4
		19		