

Figure 1

Two blocks, A and B , of masses $2m$ and $3m$ respectively, are attached to the ends of a light string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined at angle α to the horizontal ground, where $\tan \alpha = \frac{5}{12}$

The string passes over a small smooth pulley, P , fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane. Block B hangs freely below P , as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{2}{3}$

The blocks are released from rest with the string taut and A moves up the plane.

The tension in the string immediately after the blocks are released is T .

The blocks are modelled as particles and the string is modelled as being inextensible.

(a) Show that $T = \frac{12mg}{5}$

(8)

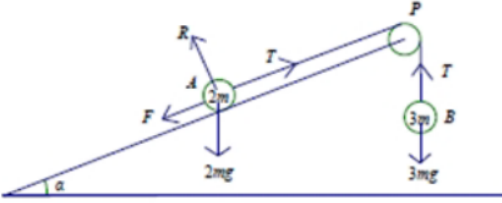
After B reaches the ground, A continues to move up the plane until it comes to rest before reaching P .

(b) Determine whether A will remain at rest, carefully justifying your answer.

(2)

(c) Suggest two refinements to the model that would make it more realistic.

(2)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)			
	$R = 2mg \cos \alpha = \frac{24mg}{13}$	B1	This mark is given for using the model to state the normal reaction between A and the plane
	$F_{\max} = \frac{2}{3} R = \frac{16mg}{13}$	B1	This mark is given for the use of $F = \mu R$
	Equation of motion for A is $T - F_{\max} - 2mg \sin \alpha = 2ma$	M1	This mark is given for a method from an equation of motion for A
		A1	This mark is given for a correct equation of motion for A
	Equation of motion for B is $3mg - T = 3ma$	M1	This mark is given for a method to form an equation of motion for B
		A1	This mark is given for a correct equation of motion for B
	$3mg - \frac{16mg}{13} - \frac{10mg}{13} = 5ma$	M1	This mark is given for a method using the equations of motion for A and B to solve for T
	$T = 3mg - \frac{3mg}{5} = \frac{12mg}{5}$	A1	This mark is given for a full method and correct working to show the answer given
(b)	$F_{\max} = \frac{16mg}{13} > \frac{10mg}{13}$ $\frac{10mg}{13}$ is the component of the weight parallel to the slope	M1	This mark is given for a comparison of F_{\max} with the component of weight
	Thus A will not move	A1	This mark is given for a fully justified and correct conclusion
(c)	Have the model consider air resistance	B1	This mark is given for one correct refinement stated
	Have the model use an extensible string	B1	This mark is given for one correct refinement stated

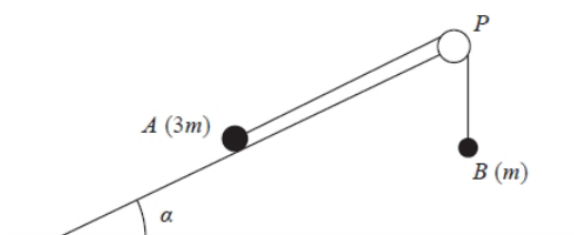


Figure 1

A small stone A of mass $3m$ is attached to one end of a string.

A small stone B of mass m is attached to the other end of the string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$

The string passes over a pulley P that is fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.

Stone B hangs freely below P , as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{1}{6}$

Stone A is released from rest and begins to move down the plane.

The stones are modelled as particles.

The pulley is modelled as being small and smooth.

The string is modelled as being light and inextensible.

Using the model for the motion of the system before B reaches the pulley,

(a) write down an equation of motion for A

(2)

(b) show that the acceleration of A is $\frac{1}{10}g$

(7)

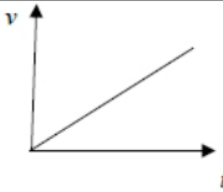
(c) sketch a velocity-time graph for the motion of B , from the instant when A is released from rest to the instant just before B reaches the pulley, explaining your answer.

(2)

In reality, the string is not light.

(d) State how this would affect the working in part (b).

(1)

Question	Scheme	Marks	AOs
	Mark parts (a) and (b) together		
(a)	Equation of motion for A	M1	3.3
	$3mg \sin \alpha - F - T = 3ma$	A1	1.1b
		(2)	
(b)	Resolve perpendicular to the plane	M1	3.4
	$R = 3mg \cos \alpha$	A1	1.1b
	$F = \frac{1}{6}R$	B1	1.2
	Equation of motion for B OR for whole system	M1	3.3
	$T - mg = ma$ OR $3mg \sin \alpha - F - mg = 3ma + ma$	A1	1.1b
	Complete method to solve for a	DM1	3.1b
	$a = \frac{1}{10}g$ *	A1*	2.2a
		(7)	
(c)		B1	1.1b
	e.g. acceleration (of B) is constant; dependent on first B1	DB1	2.4
		(2)	
(d)	e.g. the tensions in the two equations of motion would be different. Tension on A would be different to tension on B	B1	3.5a
		(1)	
(12 marks)			

3

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

A rough plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ .

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ .

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A , carefully justifying your answer.

(2)

Question	Scheme	Marks	AOs
(a)	$R = mg \cos \alpha$	B1	3.1b
	Resolve parallel to the plane	M1	3.1b
	$-F - mg \sin \alpha = -0.8mg$	A1	1.1b
	$F = \mu R$	M1	1.2
	Produce an equation in μ only and solve for μ	M1	2.2a
	$\mu = \frac{1}{4}$	A1	1.1b
		(6)	
(b)	Compare $\mu mg \cos \alpha$ with $mg \sin \alpha$	M1	3.1b
	Deduce an appropriate conclusion	A1 ft	2.2a
		(2)	
(8 marks)			
Notes:			
<p>(a)</p> <p>B1: for $R = mg \cos \alpha$</p> <p>1st M1: for resolving parallel to the plane</p> <p>1st A1: for a correct equation</p> <p>2nd M1: for use of $F = \mu R$</p> <p>3rd M1: for eliminating F and R to give a value for μ</p> <p>2nd A1: for $\mu = \frac{1}{4}$</p>			
<p>(b)</p> <p>M1: comparing size of limiting friction with weight component down the plane</p> <p>A1ft: for an appropriate conclusion from their values</p>			

Unless otherwise stated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

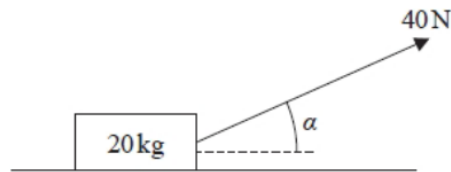


Figure 1

A wooden crate of mass 20 kg is pulled in a straight line along a rough horizontal floor using a handle attached to the crate.

The handle is inclined at an angle α to the floor, as shown in Figure 1, where $\tan \alpha = \frac{3}{4}$

The tension in the handle is 40 N.

The coefficient of friction between the crate and the floor is 0.14

The crate is modelled as a particle and the handle is modelled as a light rod.

Using the model,

(a) find the acceleration of the crate.

(6)

The crate is now pushed along the same floor using the handle. The handle is again inclined at the same angle α to the floor, and the thrust in the handle is 40 N as shown in Figure 2 below.

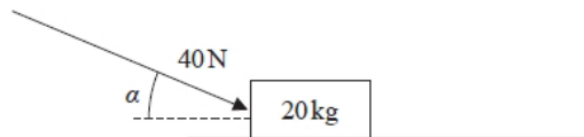


Figure 2

(b) Explain briefly why the acceleration of the crate would now be less than the acceleration of the crate found in part (a).

(2)

Question	Scheme	Marks	AOs
(a)	Resolve vertically	M1	3.1b
	$R + 40 \sin \alpha = 20g$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$40 \cos \alpha - F = 20a$	A1	1.1b
	$F = 0.14R$	B1	1.2
	$a = 0.396$ or $0.40 \text{ (m s}^{-2}\text{)}$	A1	2.2a
		(6)	
(b)	Pushing will increase R which will increase available F	B1	2.4
	Increasing F will <u>decrease</u> a * GIVEN ANSWER	B1*	2.4
		(2)	
(8 marks)			
Notes:			
<p>(a)</p> <p>M1: Resolve vertically with usual rules applying</p> <p>A1: Correct equation. Neither g nor $\sin \alpha$ need to be substituted</p> <p>M1: Apply $F = ma$ horizontally, with usual rules</p> <p>A1: Neither F nor $\cos \alpha$ need to be substituted</p> <p>B1: $F = 0.14R$ seen (e.g. on a diagram)</p> <p>A1: Either answer</p>			
<p>(b)</p> <p>B1: Pushing increases R which produces an increase in available (limiting) friction</p> <p>B1: F increase produces an <u>a decrease</u> (need to see this)</p> <p>N.B. It is possible to score B0 B1 but for the B1, some "explanation" is needed to say why friction is increased e.g. by pushing into the ground.</p>			

5

A rough plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$

A brick P of mass m is placed on the plane.

The coefficient of friction between P and the plane is μ

Brick P is in equilibrium and on the point of sliding down the plane.

Brick P is modelled as a particle.

Using the model,

(a) find, in terms of m and g , the magnitude of the normal reaction of the plane on brick P (2)

(b) show that $\mu = \frac{3}{4}$ (4)

For parts (c) and (d), you are not required to do any further calculations.

Brick P is now removed from the plane and a much heavier brick Q is placed on the plane.

The coefficient of friction between Q and the plane is also $\frac{3}{4}$

(c) Explain briefly why brick Q will remain at rest on the plane. (1)

Brick Q is now projected with speed 0.5 m s^{-1} down a line of greatest slope of the plane.

Brick Q is modelled as a particle.

Using the model,

(d) describe the motion of brick Q , giving a reason for your answer. (2)

Question	Scheme	Marks	AOs
(a)	Resolve perpendicular to the plane	M1	3.4
	$R = mg \cos \alpha = \frac{4}{5}mg$	A1	1.1b
		(2)	
(b)	Resolve parallel to the plane or horizontally or vertically	M1	3.4
	$F = mg \sin \alpha$ or $R \sin \alpha = F \cos \alpha$	A1	1.1b
	Use $F = \mu R$ and solve for μ	M1	2.1
	$\mu = \frac{3}{4}$ *	A1*	2.2a
		(4)	
(c)	The forces acting on Q will still balance as the m 's cancel oe Other possibilities: e.g. the <u>friction</u> will increase <u>in the same proportion</u> as <u>the weight component or force down the plane</u> . The <u>force pulling the brick down the plane</u> increases <u>by the same amount</u> as the <u>friction</u> oe This mark can be scored if they do the calculation.	B1	2.4
		(1)	
(d)	Brick Q slides down the plane with constant speed.	B1	2.4
	No resultant force down the plane (so no acceleration) oe	B1	2.4
	These marks can be scored if they do the calculation.	(2)	
(9 marks)			

Notes:		
a	M1	Correct no. of terms, condone sin/cos confusion
	A1	cao with no wrong working seen. $mg \cos 36.86$ is A0
b	M1	Correct no. of terms, condone sin/cos confusion
	A1	Correct equation
	M1	Must use $F = \mu R$ (not merely state it) to obtain a numerical value for μ . This is an independent M mark.
	A1*	Given answer correctly obtained
c	B1	Must have the 3 underlined phrases/word oe
d	B1	Must say constant speed.
	B1	Any appropriate equivalent statement