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Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Mathematics

Advanced
Paper 2: Pure Mathematics 2

Sample Assessment Material for first teaching September 2017
Time: 2 hours

Paper Reference
9MA0/02

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

(Total for Question 1 is 3 marks)

2. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos\theta = 2\sin\theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos\theta = 2\sin\theta$ $\tan\theta = 2$ $\theta = 63.4°$	$\cos\theta = 2\sin\theta$ $\cos^2\theta = 4\sin^2\theta$ $1 - \sin^2\theta = 4\sin^2\theta$ $\sin^2\theta = \frac{1}{5}$ $\sin\theta = \pm\frac{1}{\sqrt{5}}$ $\theta = \pm 26.6°$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

(Total for Question 2 is 3 marks)

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3. Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n(Ax + B)$$

where n, A and B are constants to be found.

(4)

(Total for Question 3 is 4 marks)

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4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0,\ x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

(Total for Question 4 is 5 marks)

5. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

(2)

(Total for Question 5 is 4 marks)

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6. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. **(2)**				
(ii) If $ax > b$ then $x > \frac{b}{a}$ **(2)**				
(iii) The difference between consecutive square numbers is odd. **(2)**				

(Total for Question 6 is 6 marks)

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7. (a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

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8.

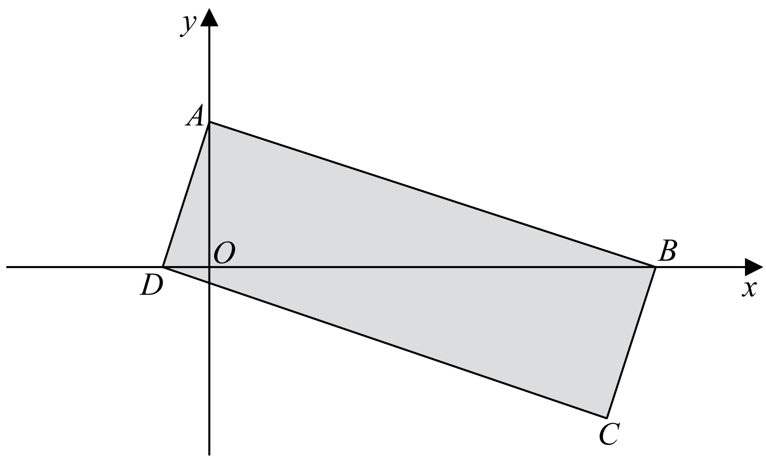

Figure 1

Figure 1 shows a rectangle *ABCD*.

The point *A* lies on the *y*-axis and the points *B* and *D* lie on the *x*-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$

(a) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle *ABCD*.

(3)

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9. Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right)\mathrm{d}x = 2A^2$$

show that there are exactly two possible values for A.

(5)

(Total for Question 9 is 5 marks)

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10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\dfrac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

(Total for Question 10 is 4 marks)

11.

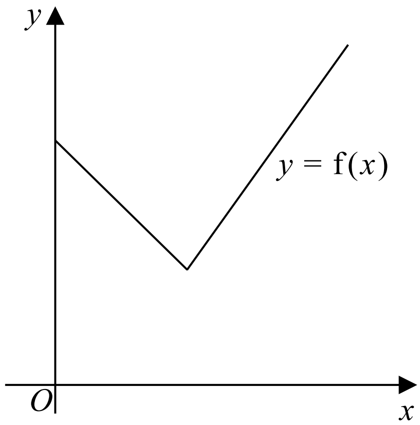

Figure 2

Figure 2 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$\mathrm{f}(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

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12. (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

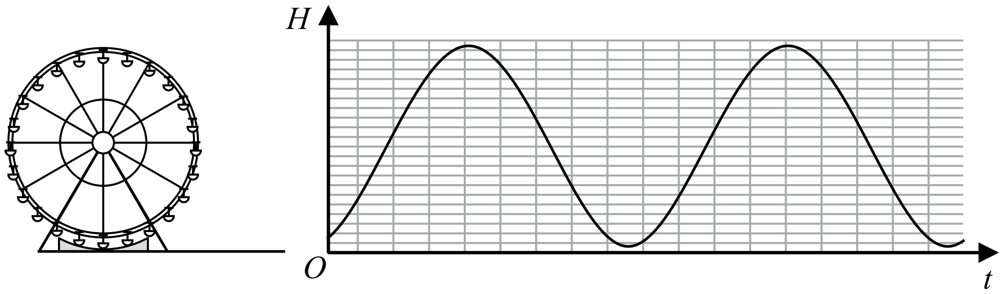

Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

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15.

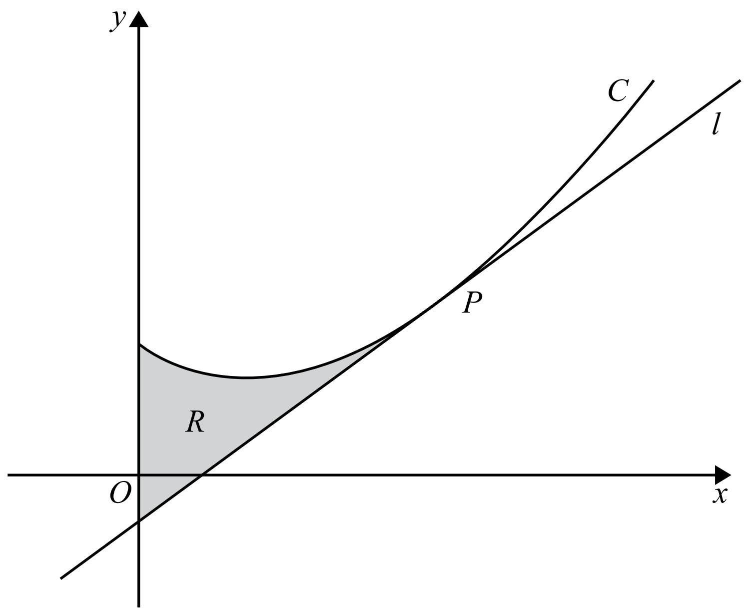

Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, \ x \geqslant 0$$

The point P with coordinates (4, 15) lies on C.

The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

16. (a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11 - 2P), \quad t \geqslant 0, \qquad 0 < P < 5.5$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + Ce^{-\frac{1}{2}t}}$$

where A, B and C are integers to be found.

(3)

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Paper 2: Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1	Sets $f(-2)=0 \Rightarrow 2\times(-2)^3-5\times(-2)^2+a\times-2+a=0$	M1	3.1a
	Solves linear equation $2a-a=-36 \Rightarrow a=$	dM1	1.1b
	$\Rightarrow a=-36$	A1	1.1b
			(3 marks)

Notes:

M1: Selects a suitable method given that $(x+2)$ is a factor of $f(x)$
Accept either setting $f(-2)=0$ or attempted division of $f(x)$ by $(x+2)$

dM1: Solves linear equation in a. Minimum requirement is that there are two terms in 'a' which must be collected to get $..a=..\Rightarrow a=$

A1: $a=-36$

Question	Scheme	Marks	AOs
2(a)	Identifies an error for student A: They use $\dfrac{\cos\theta}{\sin\theta}=\tan\theta$ It should be $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6°)\neq 2\sin(-26.6°)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
			(3 marks)

Notes:

(a)

B1: Accept a response of the type 'They use $\dfrac{\cos\theta}{\sin\theta}=\tan\theta$. This is incorrect as $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta=\dfrac{1}{2}$ not $\tan\theta=2$'

Accept also statements such as 'it should be $\cot\theta=2$'

(b)

B1: Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta=2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°)=+ve$ and $2\sin(-26.6°)=-ve$ and stating that they therefore cannot be equal.

B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x=5$ squared gives $x^2=25$ which has answers ± 5

Question	Scheme	Marks	AOs
3	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\frac{dy}{dx} = (2x+1)^3\{(2x+1)+8x\}$	M1	1.1b
	$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$	A1	1.1b
			(4 marks)

Notes:

M1: Applies the product rule to reach $\frac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$

A1: $\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$

M1: Takes out a common factor of $(2x+1)^3$

A1: The form of this answer is given. Look for $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$

Question	Scheme	Marks	AOs
4 (a)	$\text{gf}(x) = 3\ln e^{x}$	M1	1.1b
	$= 3x,\ \left(x \in \mathbb{R}\right)$	A1	1.1b
		(2)	
(b)	$\text{gf}(x) = \text{fg}(x) \Rightarrow 3x = x^{3}$	M1	1.1b
	$\Rightarrow x^{3} - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
			(5 marks)

Notes:

(a)

M1: For applying the functions in the correct order

A1: The simplest form is required so it must be $3x$ and not left in the form $3\ln e^{x}$
An answer of $3x$ with no working would score both marks

(b)

M1: Allow the candidates to score this mark if they have $e^{3\ln x} =$ their $3x$

M1: For solving their cubic in x and obtaining at least one solution.

A1: For either stating that $x = \sqrt{3}$ **only** as $\ln x$ (or $3\ln x$) is not defined at $x = 0$ and $-\sqrt{3}$
or stating that $3x = x^{3}$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3\ln x$) is not defined for $x \leqslant 0$ so therefore there is only one (real) answer.

Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)

Question	Scheme	Marks	AOs
5(a)	Substitutes $t = 0.5$ into $m = 25\mathrm{e}^{-0.05t} \Rightarrow m = 25\mathrm{e}^{-0.05\times 0.5}$	M1	3.4
	$\Rightarrow m = 24.4\text{g}$	A1	1.1b
		(2)	
(b)	States or uses $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-0.05t}\right) = \pm C\,\mathrm{e}^{-0.05t}$	M1	2.1
	$\frac{\mathrm{d}m}{\mathrm{d}t} = -0.05\times 25\mathrm{e}^{-0.05t} = -0.05m \Rightarrow k = -0.05$	A1	1.1b
		(2)	
		(4 marks)	

Notes:

(a)

M1: Substitutes $t = 0.5$ into $m = 25\mathrm{e}^{-0.05t} \Rightarrow m = 25\mathrm{e}^{-0.05\times 0.5}$

A1: $m = 24.4\text{g}$ An answer of $m = 24.4\text{g}$ with no working would score both marks

(b)

M1: Applies the rule $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{kx}\right) = k\,\mathrm{e}^{kx}$ in this context by stating or using $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-0.05t}\right) = \pm C\,\mathrm{e}^{-0.05t}$

A1: $\frac{\mathrm{d}m}{\mathrm{d}t} = -0.05\times 25\mathrm{e}^{-0.05t} = -0.05m \Rightarrow k = -0.05$

Question	Scheme	Marks	AOs
6(i)	$x^2-6x+10=(x-3)^2+1$	M1	2.1
	Deduces "always true" as $(x-3)^2\geqslant 0 \Rightarrow (x-3)^2+1\geqslant 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a>0$ then $ax>b \Rightarrow x>\frac{b}{a}$ if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $=(n+1)^2-n^2=2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1$ = (even +1) = odd	A1	2.2a
		(2)	
		(6 marks)	

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y=x^2-6x+10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$ or simply $-3x>6 \Rightarrow x<-2$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.
For example by attempting $(n+1)^2-n^2=2n+1$ or $m^2-n^2=(m-n)(m+n)$ with $m=n+1$

A1: States always true with reason and proof
Accept a proof written in words. For example
If integers are consecutive, one is odd and one is even
When squared $\text{odd}\times\text{odd}=\text{odd}$ and $\text{even}\times\text{even}=\text{even}$
The difference between odd and even is always odd, hence always true
Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

Question	Scheme	Marks	AOs
7(a)	$\sqrt{(4-x)}=2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}=1+\frac{1}{2}\left(-\frac{1}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^{2}+\ldots$	M1	1.1b
	$\sqrt{(4-x)}=2\left(1-\frac{1}{8}x-\frac{1}{128}x^{2}+..\right)$	A1	1.1b
	$\sqrt{(4-x)}=2-\frac{1}{4}x-\frac{1}{64}x^{2}+\ldots$ and $k=-\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $\lvert x\rvert<4,$ so $x=1$ can be used	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Takes out a factor of 4 and writes $\sqrt{(4-x)}=2(1\pm\ldots)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion with $n=\frac{1}{2}$

Eg. $(1+ax)^{\frac{1}{2}}=1+\frac{1}{2}(ax)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^{2}+\ldots$

A1: Correct expression inside the bracket $1-\frac{1}{8}x-\frac{1}{128}x^{2}+$ which may be left unsimplified

A1: $\sqrt{(4-x)}=2-\frac{1}{4}x-\frac{1}{64}x^{2}+\ldots$ and $k=-\frac{1}{64}$

(b)

B1: The expansion is valid for $\lvert x\rvert<4,$ so $x=1$ can be used

Question	Scheme	Marks	AOs
8 (a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2+2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2+2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	
			(7 marks)

Notes:

(a) It is important that the student communicates each of these steps clearly

B1: States the gradient of AB is $-\frac{2}{5}$

B1: States that y coordinate of $A = 2$

M1: Uses the form $y = mx + c$ with $m =$ their adapted $-\frac{2}{5}$ and $c =$ their 2

Alternatively uses the form $y - y_1 = m(x - x_1)$ with $m =$ their adapted $-\frac{2}{5}$ and $(x_1, y_1) = (0, 2)$

A1*: Proceeds to given answer

(b)

M1: Finds the lengths of AB or AD using Pythagoras' Theorem. Look for $\sqrt{5^2+2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2+2^2}$

Alternatively finds the lengths BD and AO using coordinates. Look for $\left(5+\frac{4}{5}\right)$ and 2

M1: For a full method of finding the area of the rectangle $ABCD$. Allow for $AD \times AB$

Alternatively attempts area $ABCD = 2 \times \frac{1}{2} BD \times AO = 2 \times \frac{1}{2}\, '5.8' \times '2'$

A1: Area $ABCD = 11.6$ or other exact equivalent such as $\frac{58}{5}$

Question	Scheme		Marks	AOs
9	$\int(3x^{0.5}+A)\,\mathrm{d}x = 2x^{1.5}+Ax(+c)$		M1 A1	3.1a 1.1b
	Uses limits and sets $= 2A^2 \Rightarrow (2\times 8+4A)-(2\times 1+A)=2A^2$		M1	1.1b
	Sets up quadratic and attempts to solve	Sets up quadratic and attempts b^2-4ac	M1	1.1b
	$\Rightarrow A=-2, \frac{7}{2}$ **and** states that there are two roots	States $b^2-4ac=121>0$ and hence there are two roots	A1	2.4
			(5 marks)	

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1.5}+Ax(+c)$ where k is a non- zero constant

A1: Correct answer but may not be simplified

M1: Substitutes in limits and subtracts. This can only be scored if $\int A\,\mathrm{d}x = Ax$ and not $\frac{A^2}{2}$

M1: Sets up quadratic equation in A and either attempts to solve or attempts b^2-4ac

A1: Either $A=-2, \frac{7}{2}$ **and** states that there are two roots

Or states $b^2-4ac=121>0$ and hence there are two roots

Question	Scheme	Marks	AOs
10	Attempts $S_\infty = \frac{8}{7}\times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7}\times\frac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1 = \frac{8}{7}\times(1-r^6)$	M1	2.1
	$\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = ..$	M1	1.1b
	$\Rightarrow r = \pm\frac{1}{\sqrt{2}}$ (so $k=2$)	A1	1.1b
		(4 marks)	

Notes:

M1: Substitutes the correct formulae for S_∞ and S_6 into the given equation $S_\infty = \frac{8}{7}\times S_6$

M1: Proceeds to an equation just in r

M1: Solves using a correct method

A1: Proceeds to $r=\pm\frac{1}{\sqrt{2}}$ giving $k=2$

Question	Scheme	Marks	AOs
11 (a)	$\mathrm{f}(x) \geqslant 5$	B1	1.1b
		(1)	
(b)	Uses $-2(3-x)+5=\frac{1}{2}x+30$	M1	3.1a
	Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x=31$	M1	1.1b
	$x=\frac{62}{3}$ only	A1	1.1b
		(3)	
(c)	Makes the connection that there must be two intersections. Implied by either end point $k>5$ or $k \leqslant 11$	M1	2.2a
	$\{k : k \in \mathbb{R}, 5<k \leqslant 11\}$	A1	2.5
		(2)	
		(6 marks)	

Notes:

(a)

B1: $\mathrm{f}(x) \geqslant 5$ Also allow $\mathrm{f}(x) \in [5, \infty)$

(b)

M1: Deduces that the solution to $\mathrm{f}(x)=\frac{1}{2}x+30$ can be found by solving

$-2(3-x)+5=\frac{1}{2}x+30$

M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms

A1: $x=\frac{62}{3}$ only. Do not allow 20.6

(c)

M1: Deduces that two distinct roots occurs when $y=k$ intersects $y=\mathrm{f}(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k \leqslant 11$

A1: Correct solution only $\{k : k \in \mathbb{R}, 5<k \leqslant 11\}$

Question	Scheme	Marks	AOs
12(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9\left(1 - \sin^2 x\right)$	M1	3.1a
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1b
	$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$	M1	1.1b
	$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$	A1	1.1b
		(6)	
(b)	Attempts $2\theta - 30° = -19.47°$	M1	3.1a
	$\Rightarrow \theta = 5.26°$	A1ft	1.1b
		(2)	
			(8 marks)

Notes:

(a)

M1: Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$

A1: $12\sin^2 x + \sin x - 1 = 0$ or exact equivalent

M1: Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

A1: $\sin x = \frac{1}{4}, -\frac{1}{3}$

M1: Obtains two correct values for their $\sin x = k$

A1: All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$

(b)

M1: For setting $2\theta - 30° = \text{their}\ '-19.47°'$

A1ft: $\theta = 5.26°$ but allow a follow through on their $'-19.47°'$

Question	Scheme	Marks	AOs
13(a)	$R=\sqrt{109}$	B1	1.1b
	$\tan\alpha=\frac{3}{10}$	M1	1.1b
	$\alpha=16.70°$ so $\sqrt{109}\cos(\theta+16.70°)$	A1	1.1b
		(3)	
(b)	(i) e.g $H=11-10\cos(80t)°+3\sin(80t)°$ or $H=11-\sqrt{109}\cos(80t+16.70)°$	B1	3.3
	(ii) $11+\sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t+"16.70"=540$	M1	3.4
	$t=\frac{540-"16.70"}{80}=(6.54)$	M1	1.1b
	t = 6 mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H=11-10\cos(90t)°+3\sin(90t)°$		3.3
		(1)	
			(9 marks)

Notes:

(a)

B1: $R=\sqrt{109}$ Do not allow decimal equivalents

M1: Allow for $\tan\alpha=\pm\frac{3}{10}$

A1: $\alpha=16.70°$

(b)(i)

B1: see scheme

(b)(ii)

B1ft: their $11+$ their $\sqrt{109}$ Allow decimals here.

(c)

M1: Sets $80t+"16.70"=540$. Follow through on their 16.70

M1: Solves their $80t+"16.70"=540$ correctly to find t

A1: t = 6 mins 32 seconds

(d)

B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.

Question	Scheme	Marks	AOs
14(a)	Sets $500 = \pi r^2 h$	B1	2.1
	Substitute $h = \dfrac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$	M1	2.1
	Simplifies to reach given answer $S = 2\pi r^2 + \dfrac{1000}{r}$ *	A1*	1.1b
		(3)	
(b)	Differentiates S with both indices correct in $\dfrac{dS}{dr}$	M1	3.4
	$\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$	A1	1.1b
	Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant	M1	2.1
	Radius $= 4.30$ cm	A1	1.1b
	Substitutes their $r = 4.30$ into $h = \dfrac{500}{\pi r^2} \Rightarrow$ Height $= 8.60$ cm	A1	1.1b
		(5)	
(c)	States a valid reason such as • The radius is too big for the size of our hands • If $r = 4.3$ cm and $h = 8.6$ cm the can is square in profile. All drinks cans are taller than they are wide • The radius is too big for us to drink from • They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans	B1	3.2a
		(1)	
			9 marks

Notes:

(a)

B1: Uses the correct volume formula with V=500. Accept $500 = \pi r^2 h$

M1: Substitutes $h = \dfrac{500}{\pi r^2}$ or $rh = \dfrac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi r h$ to get S as a function of r

A1*: $S = 2\pi r^2 + \dfrac{1000}{r}$ Note that this is a given answer.

(b)

M1: Differentiates the given S to reach $\dfrac{dS}{dr} = Ar \pm Br^{-2}$

A1: $\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$ or exact equivalent

M1: Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant

A1: R = awrt 4.30cm

A1: H = awrt 8.60 cm

(c)

B1: Any valid reason. See scheme for alternatives

Question	Scheme	Marks	AOs
15	$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$	M1 A1	3.1a 1.1b
	Substitutes $x = 4 \Rightarrow \frac{dy}{dx} = 6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y - 15 = 6(x - 4)$	M1	2.1
	Equation of l is $y = 6x - 9$	A1	1.1b
	Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x - 9)\,dx$	M1	3.1a
	$= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c)\right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x\right]_0^4 = 2 \times 4^{\frac{5}{2}} - \frac{15}{2} \times 4^2 + 20 \times 4 - 0$	M1	2.1
	Area of $R = 24$ *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
		(10)	
			(10 marks)

Question 15 continued
Notes:

M1: Differentiates $5x^{\frac{3}{2}}-9x+11$ to a form $Ax^{\frac{1}{2}}+B$

A1: $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{15}{2}x^{\frac{1}{2}}-9$ but may not be simplified

M1: Substitutes $x=4$ in their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ to find the gradient of the tangent

M1: Uses their gradient and the point (4, 15) to find the equation of the tangent

A1: Equation of l is $y = 6x - 9$

M1: Uses Area $R = \displaystyle\int_0^4 \left(5x^{\frac{3}{2}}-9x+11\right)-\left(6x-9\right)\mathrm{d}x$ following through on their $y=6x-9$

Look for a form $Ax^{\frac{5}{2}}+Bx^2+Cx$

A1: $=\left[2x^{\frac{5}{2}}-\dfrac{15}{2}x^2+20x\left(+c\right)\right]_0^4$ This must be correct but may not be simplified

M1: Substitutes in both limits and subtracts

A1*: Correct area for $R = 24$

A1: Uses correct notation and produces a well explained and accurate solution. Look for

- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of l. See scheme.
- Correct explanation in finding the area of R. In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

M1: Area under curve = $\displaystyle\int_0^4 \left(5x^{\frac{3}{2}}-9x+11\right)=\left[Ax^{\frac{5}{2}}+Bx^2+Cx\right]_0^4$

A1: $=\left[2x^{\frac{5}{2}}-\dfrac{9}{2}x^2+11x\right]_0^4=36$

M1: This requires a full method with all triangles found using a correct method

Look for Area $R=$ their $36-\dfrac{1}{2}\times 15\times\left(4-\text{their }\dfrac{3}{2}\right)+\dfrac{1}{2}\times\text{their }9\times\text{their }\dfrac{3}{2}$

Question	Scheme	Marks	AOs
16(a)	Sets $\dfrac{1}{P(11-2P)} = \dfrac{A}{P} + \dfrac{B}{(11-2P)}$	B1	1.1a
	Substitutes either $P=0$ or $P=\dfrac{11}{2}$ into $1 = A(11-2P)+BP \Rightarrow A \text{ or } B$	M1	1.1b
	$\dfrac{1}{P(11-2P)} = \dfrac{1/11}{P} + \dfrac{2/11}{(11-2P)}$	A1	1.1b
		(3)	
(b)	Separates the variables $\displaystyle\int \frac{22}{P(11-2P)}\,\mathrm{d}P = \int 1\,\mathrm{d}t$	B1	3.1a
	Uses (a) and attempts to integrate $\displaystyle\int \frac{2}{P} + \frac{4}{(11-2P)}\,\mathrm{d}P = t + c$	M1	1.1b
	$2\ln P - 2\ln(11-2P) = t + c$	A1	1.1b
	Substitutes $t=0, P=1 \Rightarrow t=0, P=1 \Rightarrow c = (-2\ln 9)$	M1	3.1a
	Substitutes $P=2 \Rightarrow t = 2\ln 2 + 2\ln 9 - 2\ln 7$	M1	3.1a
	Time $= 1.89$ years	A1	3.2a
		(6)	
(c)	Uses ln laws $2\ln P - 2\ln(11-2P) = t - 2\ln 9$ $\Rightarrow \ln\left(\dfrac{9P}{11-2P}\right) = \dfrac{1}{2}t$	M1	2.1
	Makes 'P' the subject $\Rightarrow \left(\dfrac{9P}{11-2P}\right) = \mathrm{e}^{\frac{1}{2}t}$ $\Rightarrow 9P = (11-2P)\mathrm{e}^{\frac{1}{2}t}$ $\Rightarrow P = \mathrm{f}\left(\mathrm{e}^{\frac{1}{2}t}\right)$ or $\Rightarrow P = \mathrm{f}\left(\mathrm{e}^{-\frac{1}{2}t}\right)$	M1	2.1
	$\Rightarrow P = \dfrac{11}{2+9\mathrm{e}^{-\frac{1}{2}t}} \Rightarrow A = 11, B = 2, C = 9$	A1	1.1b
		(3)	
		(12 marks)	

Question 16 continued

Notes:

(a)

B1: Sets $\dfrac{1}{P(11-2P)}=\dfrac{A}{P}+\dfrac{B}{(11-2P)}$

M1: Substitutes $P=0$ or $P=\dfrac{11}{2}$ into $1=A(11-2P)+BP \Rightarrow A \text{ or } B$

Alternatively compares terms to set up and solve two simultaneous equations in A and B

A1: $\dfrac{1}{P(11-2P)}=\dfrac{1/11}{P}+\dfrac{2/11}{(11-2P)}$ or equivalent $\dfrac{1}{P(11-2P)}=\dfrac{1}{11P}+\dfrac{2}{11(11-2P)}$

Note: The correct answer with no working scores all three marks.

(b)

B1: Separates the variables to reach $\displaystyle\int \frac{22}{P(11-2P)}\,\mathrm{d}P=\int 1\,\mathrm{d}t$ or equivalent

M1: Uses part (a) and $\displaystyle\int \frac{A}{P}+\frac{B}{(11-2P)}\mathrm{d}P = A\ln P \pm C\ln(11-2P)$

A1: Integrates both sides to form a correct equation including a 'c' Eg
$2\ln P-2\ln(11-2P)=t+c$

M1: Substitutes $t = 0$ and $P = 1$ to find c

M1: Substitutes $P=2$ to find t. This is dependent upon having scored both previous M's

A1: Time = 1.89 years

(c)

M1: Uses correct log laws to move from $2\ln P-2\ln(11-2P)=t+c$ to $\ln\left(\dfrac{P}{11-2P}\right)=\dfrac{1}{2}t+d$ for their numerical 'c'

M1: Uses a correct method to get P in terms of $\mathrm{e}^{\frac{1}{2}t}$

This can be achieved from $\ln\left(\dfrac{P}{11-2P}\right)=\dfrac{1}{2}t+d \Rightarrow \dfrac{P}{11-2P}=\mathrm{e}^{\frac{1}{2}t+d}$ followed by cross multiplication and collection of terms in P (See scheme)

Alternatively uses a correct method to get P in terms of $\mathrm{e}^{-\frac{1}{2}t}$ For example
$\dfrac{P}{11-2P}=\mathrm{e}^{\frac{1}{2}t+d} \Rightarrow \dfrac{11-2P}{P}=\mathrm{e}^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \dfrac{11}{P}-2=\mathrm{e}^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \dfrac{11}{P}=2+\mathrm{e}^{-\left(\frac{1}{2}t+d\right)}$ followed by division

A1: Achieves the correct answer in the form required. $P=\dfrac{11}{2+9\mathrm{e}^{-\frac{1}{2}t}} \Rightarrow A=11, B=2, C=9$ oe