Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Mathematics

Advanced
Paper 1: Pure Mathematics 1

Sample Assessment Material for first teaching September 2017

Time: 2 hours

Paper Reference

9MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(ii) $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

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2.

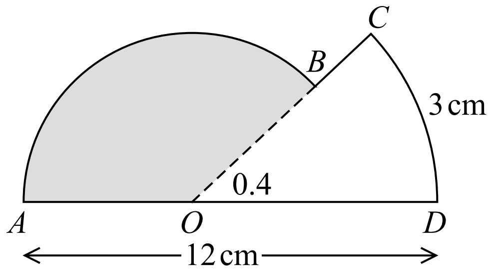

Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, $\angle COD = 0.4$ radians and *AOD* is a straight line of length 12 cm,

(a) find the length of *OD*,

(2)

(b) find the area of the shaded sector *AOB*.

(3)

(Total for Question 2 is 5 marks)

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3. A circle C has equation

$$x^2 + y^2 - 4x + 10y = k$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

(b) State the range of possible values for k.

(2)

(Total for Question 3 is 4 marks)

4. Given that a is a positive constant and

$$\int_{a}^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

(Total for Question 4 is 4 marks)

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5. A curve C has parametric equations

$$x = 2t - 1, \qquad y = 4t - 7 + \frac{3}{t}, \qquad t \neq 0$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \qquad x \neq -1$$

where a and b are integers to be found.

(3)

(Total for Question 5 is 3 marks)

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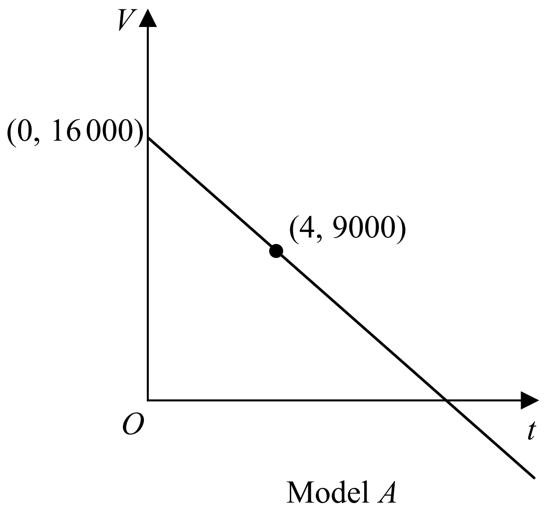
6. A company plans to extract oil from an oil field.

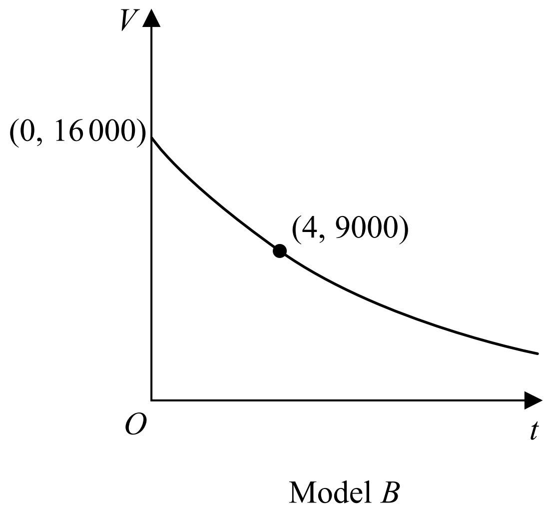
The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.





(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

7.

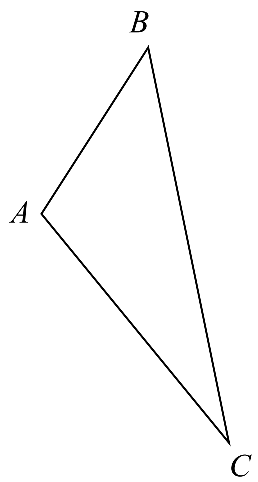


Figure 2

Figure 2 shows a sketch of a triangle ABC.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

(5)

8. $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval $[3.5, 4]$

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

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9. (a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

(Total for Question 9 is 5 marks)

10. Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$

(5)

(Total for Question 10 is 5 marks)

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11. An archer shoots an arrow.

The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.

(3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.

(1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where A, B and C are constants to be found.

(3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

Hence or otherwise, find, for the adapted model

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.

(2)

12. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

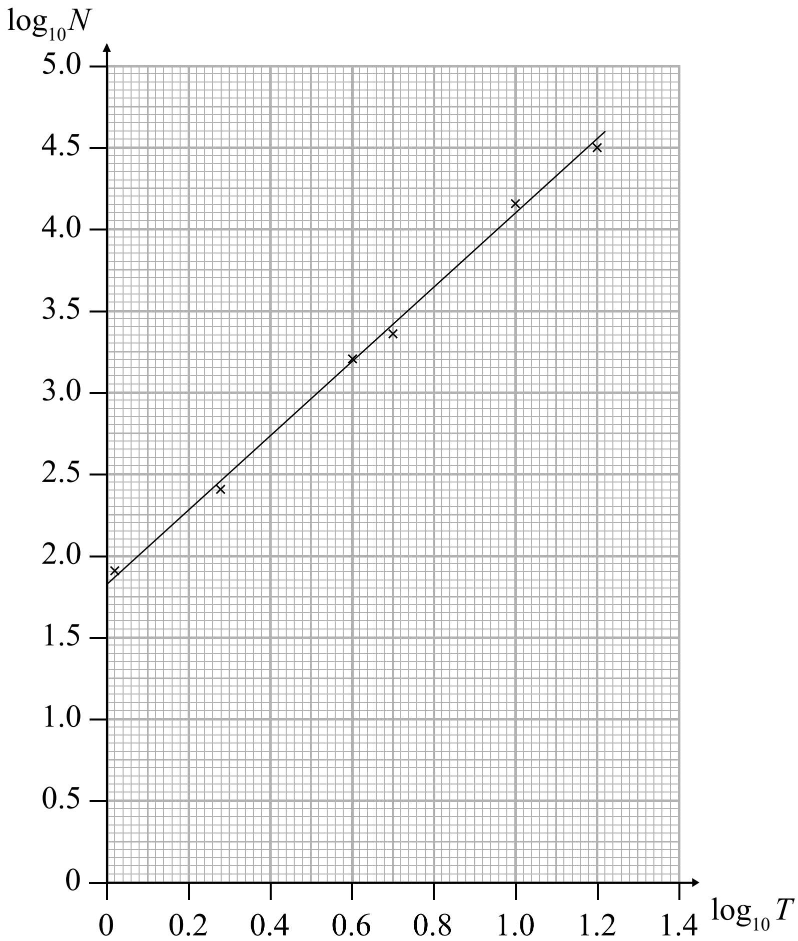

Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

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13. The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(2)

The point P lies on C where $t = \dfrac{2\pi}{3}$

The line l is the normal to C at P.

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.

You must show clearly how you obtained your answers.

(6)

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14.

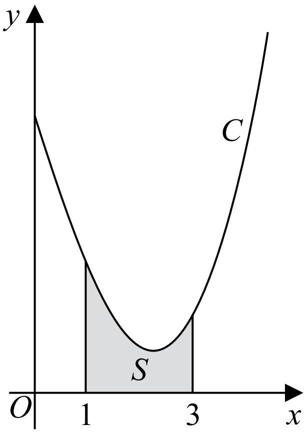

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

(In part c, solutions based entirely on graphical or numerical methods are not acceptable.)

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15.

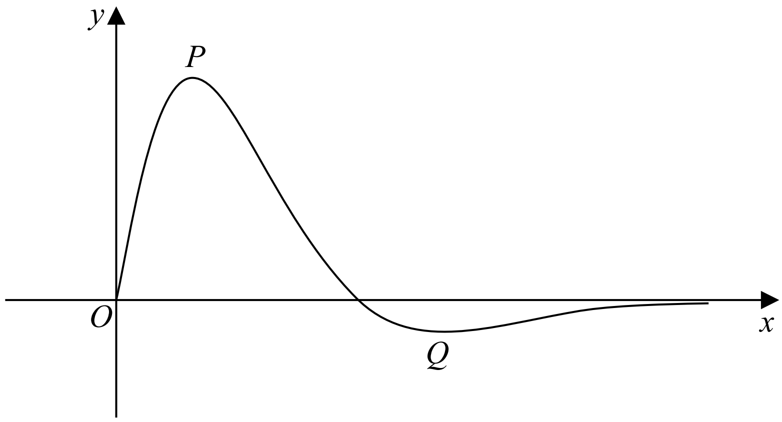


Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{\mathrm{e}^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$.

(ii) $y = 3 - 2\mathrm{f}(x)$.

(4)

Paper 1: Pure Mathematics 1 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	(i) $\frac{dy}{dx} = 12x^3 - 24x^2$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 36x^2 - 48x$	A1ft	1.1b
		(3)	
(b)	Substitutes $x = 2$ into their $\frac{dy}{dx} = 12\times 2^3 - 24\times 2^2$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point"	A1	2.1
		(2)	
(c)	Substitutes $x = 2$ into their $\frac{d^2y}{dx^2} = 36\times 2^2 - 48\times 2$	M1	1.1b
	$\frac{d^2y}{dx^2} = 48 > 0$ and states "hence the stationary point is a minimum"	A1ft	2.2a
		(2)	
			(7 marks)

Notes:

(a)(i)

M1: Differentiates to a cubic form

A1: $\frac{dy}{dx} = 12x^3 - 24x^2$

(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 36x^2 - 48x$

(b)

M1: Substitutes $x = 2$ into their $\frac{dy}{dx}$

A1: Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" All aspects of the proof must be correct

(c)

M1: Substitutes $x = 2$ into their $\frac{d^2y}{dx^2}$

Alternatively calculates the gradient of C either side of $x = 2$

A1ft: For a correct calculation, a valid reason and a correct conclusion.

Follow through on an incorrect $\frac{d^2y}{dx^2}$

Question	Scheme	Marks	AOs
2(a)	Uses $s = r\theta \Rightarrow 3 = r \times 0.4$	M1	1.2
	$\Rightarrow OD = 7.5$ cm	A1	1.1b
		(2)	
(b)	Uses angle $AOB = (\pi - 0.4)$ or uses radius is (12 – '7.5') cm	M1	3.1a
	Uses area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times (12 - 7.5)^2 \times (\pi - 0.4)$	M1	1.1b
	$= 27.8\text{cm}^2$	A1ft	1.1b
		(3)	
			(5 marks)

Notes:

(a)

M1: Attempts to use the correct formula $s = r\theta$ with $s = 3$ and $\theta = 0.4$

A1: *OD* = 7.5 cm (An answer of 7.5cm implies the use of a correct formula and scores both marks)

(b)

M1: $AOB = \pi - 0.4$ may be implied by the use of *AOB* = awrt 2.74 or uses radius is (12 – their '7.5')

M1: Follow through on their radius (12 – their *OD*) and their angle

A1ft: Allow awrt 27.8 cm^2. (Answer 27.75862562). Follow through on their (12 – their '7.5')
Note: Do not follow through on a radius that is negative.

Question	Scheme	Marks	AOs
3(a)	Attempts $(x-2)^2+(y+5)^2=....$	M1	1.1b
	Centre (2, −5)	A1	1.1b
		(2)	
(b)	Sets $k+2^2+5^2>0$	M1	2.2a
	$\Rightarrow k>-29$	A1ft	1.1b
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts to complete the square so allow $(x-2)^2+(y+5)^2=....$

A1: States the centre is at (2, −5). Also allow written separately $x=2, y=-5$

(2, −5) implies both marks

(b)

M1: Deduces that the right hand side of their $(x\pm...)^2+(y\pm...)^2=...$ is >0 or $\geqslant 0$

A1ft: $k>-29$ Also allow $k\geqslant -29$ Follow through on their rhs of $(x\pm...)^2+(y\pm...)^2=...$

Question	Scheme	Marks	AOs
4	Writes $\int \frac{t+1}{t}\,\mathrm{d}t=\int 1+\frac{1}{t}\,\mathrm{d}t$ and attempts to integrate	M1	2.1
	$=t+\ln t\;(+c)$	M1	1.1b
	$(2a+\ln 2a)-(a+\ln a)=\ln 7$	M1	1.1b
	$a=\ln\frac{7}{2}$ with $k=\frac{7}{2}$	A1	1.1b
			(4 marks)

Notes:

M1: Attempts to divide each term by t or alternatively multiply each term by t^{-1}

M1: Integrates each term and knows $\int \frac{1}{t}\,\mathrm{d}t=\ln t$. The + c is not required for this mark

M1: Substitutes in both limits, subtracts and sets equal to ln7

A1: Proceeds to $a=\ln\frac{7}{2}$ and states $k=\frac{7}{2}$ or exact equivalent such as 3.5

Question	Scheme	Marks	AOs
5	Attempts to substitute $=\dfrac{x+1}{2}$ into $y \Rightarrow y = 4\left(\dfrac{x+1}{2}\right)-7 + \dfrac{6}{(x+1)}$	M1	2.1
	Attempts to write as a single fraction $y = \dfrac{(2x-5)(x+1)+6}{(x+1)}$	M1	2.1
	$y = \dfrac{2x^2-3x+1}{x+1}$ $\quad a=-3, b=1$	A1	1.1b
		(3 marks)	

Notes:

M1: Score for an attempt at substituting $t = \dfrac{x+1}{2}$ or equivalent into $y = 4t - 7 + \dfrac{3}{t}$

M1: Award this for an attempt at a single fraction with a correct common denominator.

Their $4\left(\dfrac{x+1}{2}\right)-7$ term may be simplified first

A1: Correct answer only $y = \dfrac{2x^2-3x+1}{x+1}$ $\quad a=-3, b=1$

Question	Scheme	Marks	AOs
6 (a)(i)	10750 barrels	B1	3.4
(ii)	Gives a valid limitation, for example • The model shows that the daily volume of oil extracted would become negative as t increases, which is impossible • States when $t=10, V=-1500$ which is impossible • States that the model will only work for $0 \leqslant t \leqslant \frac{64}{7}$	B1	3.5b
		(2)	
(b)(i)	Suggests a suitable exponential model, for example $V = Ae^{kt}$	M1	3.3
	Uses $(0,16000)$ and $(4,9000)$ in $\Rightarrow 9000 = 16000e^{4k}$	dM1	3.1b
	$\Rightarrow k = \frac{1}{4}\ln\left(\frac{9}{16}\right)$ awrt -0.144	M1	1.1b
	$V = 16000e^{\frac{1}{4}\ln\left(\frac{9}{16}\right)t}$ or $V = 16000e^{-0.144t}$	A1	1.1b
(ii)	Uses their exponential model with $t = 3 \Rightarrow V =$ awrt 10 400 barrels	B1ft	3.4
		(5)	
		(7 marks)	

Notes:

(a)(i)

B1: 10750 barrels

(a)(ii)

B1: See scheme

(b)(i)

M1: Suggests a suitable exponential model, for example $V = Ae^{kt}$, $V = Ar^t$ or any other suitable function such as $V = Ae^{kt} + b$ where the candidate chooses a value for b.

dM1: Uses both $(0,16000)$ and $(4,9000)$ in their model.

With $V = Ae^{kt}$ candidates need to proceed to $9000 = 16000e^{4k}$

With $V = Ar^t$ candidates need to proceed to $9000 = 16000r^4$

With $V = Ae^{kt} + b$ candidates need to proceed to $9000 = (16000 - b)e^{4k} + b$ where b is given as a positive constant and $A + b = 16000$.

M1: Uses a correct method to find all constants in the model.

A1: Gives a suitable equation for the model passing through (or approximately through in the case of decimal equivalents) both values $(0,16000)$ and $(4,9000)$. Possible equations for the model could be for example

$V = 16000e^{-0.144t}$ $\quad V = 16000 \times (0.866)^t$ $\quad V = 15800e^{-0.146t} + 200$

(b)(ii)

B1ft: Follow through on their exponential model

Question	Scheme	Marks	AOs
7	Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=2\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mathbf{i}-9\mathbf{j}+3\mathbf{k}=3\mathbf{i}-6\mathbf{j}+4\mathbf{k}$	M1	3.1a
	Attempts to find any one length using 3-d Pythagoras	M1	2.1
	Finds all of $\lvert AB\rvert=\sqrt{14},\quad \lvert AC\rvert=\sqrt{61},\quad \lvert BC\rvert=\sqrt{91}$	A1ft	1.1b
	$\cos BAC=\dfrac{14+61-91}{2\sqrt{14}\sqrt{61}}$	M1	2.1
	angle $BAC=105.9°$ *	A1*	1.1b
		(5)	
		(5 marks)	

Notes:

M1: Attempts to find $\overrightarrow{AC}$ by using $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

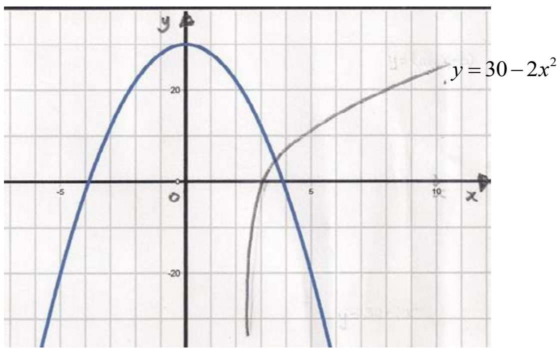
M1: Attempts to find any one length by use of Pythagoras' Theorem

A1ft: Finds all three lengths in the triangle. Follow through on their $\lvert AC\rvert$

M1: Attempts to find BAC using $\cos BAC=\dfrac{\lvert AB\rvert^2+\lvert AC\rvert^2-\lvert BC\rvert^2}{2\lvert AB\rvert\lvert AC\rvert}$

Allow this to be scored for other methods such as $\cos BAC=\dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{\lvert AB\rvert\lvert AC\rvert}$

A1*: This is a show that and all aspects must be correct. Angle $BAC=105.9°$

Question	Scheme	Marks	AOs
8 (a)	$\text{f}(3.5) = -4.8,\ \ \text{f}(4) = (+)3.1$	M1	1.1b
	Change of sign and function continuous in interval $[3.5, 4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	Attempts $x_1 = x_0 - \dfrac{\text{f}(x_0)}{\text{f}'(x_0)} \Rightarrow \quad x_1 = 4 - \dfrac{3.099}{16.67}$	M1	1.1b
	$x_1 = 3.81$	A1	1.1b
	$y = \ln(2x - 5)$	**(2)**	
(c)	 Attempts to sketch both $y = \ln(2x - 5)$ and $y = 30 - 2x^2$	M1	3.1a
	States that $y = \ln(2x - 5)$ meets $y = 30 - 2x^2$ in just one place, therefore $y = \ln(2x - 5) = 30 - 2x$ has just one root $\Rightarrow \text{f}(x) = 0$ has just one root	A1	2.4
		(2)	
			(6 marks)

Notes:

(a)

M1: Attempts f(x) at both $x = 3.5$ and $x = 4$ with at least one correct to 1 significant figure

A1*: f (3.5) and f(4) correct to 1 sig figure (rounded or truncated) with a correct reason and conclusion. A reason could be change of sign, or $\text{f}(3.5)\times\text{f}(4) < 0$ or similar with f(x) being continuous in this interval. A conclusion could be 'Hence root' or 'Therefore root in interval'

(b)

M1: Attempts $x_1 = x_0 - \dfrac{\text{f}(x_0)}{\text{f}'(x_0)}$ evidenced by $x_1 = 4 - \dfrac{3.099}{16.67}$

A1: Correct answer only $x_1 = 3.81$

(c)

M1: For a valid attempt at showing that there is only one root. This can be achieved by

- Sketching graphs of $y = \ln(2x - 5)$ and $y = 30 - 2x^2$ on the same axes
- Showing that $\text{f}(x) = \ln(2x - 5) + 2x^2 - 30$ has no turning points
- Sketching a graph of $\text{f}(x) = \ln(2x - 5) + 2x^2 - 30$

A1: Scored for correct conclusion

Question	Scheme	Marks	AOs
9(a)	$\tan\theta + \cot\theta \equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$	M1	2.1
	$\equiv \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$	A1	1.1b
	$\equiv \dfrac{1}{\frac{1}{2}\sin 2\theta}$	M1	2.1
	$\equiv 2\text{cosec}2\theta$ *	A1*	1.1b
		(4)	
(b)	States $\tan\theta + \cot\theta = 1 \Rightarrow \sin 2\theta = 2$ AND no real solutions as $-1 \leqslant \sin 2\theta \leqslant 1$	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Writes $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

A1: Achieves a correct intermediate answer of $\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

M1: Uses the double angle formula $\sin 2\theta = 2\sin\theta\cos\theta$

A1*: Completes proof with no errors. This is a given answer.

Note: There are many alternative methods. For example

$\tan\theta + \cot\theta \equiv \tan\theta + \dfrac{1}{\tan\theta} \equiv \dfrac{\tan^2\theta + 1}{\tan\theta} \equiv \dfrac{\sec^2\theta}{\tan\theta} \equiv \dfrac{1}{\cos^2\theta \times \dfrac{\sin\theta}{\cos\theta}} \equiv \dfrac{1}{\cos\theta \times \sin\theta}$ then as the main scheme.

(b)

B1: Scored for sight of $\sin 2\theta = 2$ and a reason as to why this equation has no real solutions.

Possible reasons could be $-1 \leqslant \sin 2\theta \leqslant 1$........and therefore $\sin 2\theta \neq 2$

or $\sin 2\theta = 2 \Rightarrow 2\theta = \arcsin 2$ which has no answers as $-1 \leqslant \sin 2\theta \leqslant 1$

Question	Scheme	Marks	AOs
10	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ $\Rightarrow \sin(\theta+h)=\sin\theta\cos h+\cos\theta\sin h$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$	A1	1.1b
	$=\dfrac{\sin h}{h}\cos\theta+\left(\dfrac{\cos h-1}{h}\right)\sin\theta$	M1	2.1
	Uses $h\to 0,\ \dfrac{\sin h}{h}\to 1$ and $\dfrac{\cos h-1}{h}\to 0$ Hence the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$ *	A1*	2.5
		(5 marks)	

Notes:

B1: States or implies that the gradient of the chord is $\dfrac{\sin(\theta+h)-\sin\theta}{h}$ or similar such as $\dfrac{\sin(\theta+\delta\theta)-\sin\theta}{\theta+\delta\theta-\theta}$ for a small h or $\delta\theta$

M1: Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ or $\delta\theta$

A1: Obtains $\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$ or equivalent

M1: Writes their expression in terms of $\dfrac{\sin h}{h}$ and $\dfrac{\cos h-1}{h}$

A1*: Uses correct language to explain that $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$

For this method they should use all of the given statements $h\to 0,\ \dfrac{\sin h}{h}\to 1,$ $\dfrac{\cos h-1}{h}\to 0$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$

and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$

Question	Scheme	Marks	AOs
10alt	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Sets $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\dfrac{\sin\left(\theta+\frac{h}{2}+\frac{h}{2}\right)-\sin\left(\theta+\frac{h}{2}-\frac{h}{2}\right)}{h}$ and uses the compound angle identity for $\sin(A+B)$ and $\sin(A-B)$ with $A=\theta+\dfrac{h}{2},\quad B=\dfrac{h}{2}$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=$ $\dfrac{\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)+\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]-\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)-\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]}{h}$	A1	1.1b
	$=\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\cos\left(\theta+\dfrac{h}{2}\right)$	M1	2.1
	Uses $h\to 0,\ \dfrac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ and $\cos\left(\theta+\dfrac{h}{2}\right)\to\cos\theta$ Therefore the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$ *	A1*	2.5
		(5 marks)	

Additional notes:

A1*: Uses correct language to explain that $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$. For this method they should use the (adapted) given statement $h\to 0,\ \dfrac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ with $\cos\left(\theta+\dfrac{h}{2}\right)\to\cos\theta$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$

Question	Scheme	Marks	AOs
11(a)	Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$	M1	3.4
	Solves using an appropriate method, for example $d=\dfrac{-0.4\pm\sqrt{(0.4)^2-4(-0.002)(1.8)}}{2\times-0.002}$	dM1	1.1b
	Distance $=$ awrt $204(\text{m})$ only	A1	2.2a
		(3)	
(b)	States the initial height of the arrow above the ground.	B1	3.4
		(1)	
(c)	$1.8+0.4d-0.002d^2=-0.002\left(d^2-200d\right)+1.8$	M1	1.1b
	$=-0.002\left((d-100)^2-10000\right)+1.8$	M1	1.1b
	$=21.8-0.002(d-100)^2$	A1	1.1b
		(3)	
(d)	(i) 22.1 metres	B1ft	3.4
	(ii) 100 metres	B1ft	3.4
		(2)	
		(9 marks)	

Notes:

(a)

M1: Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$

M1: Solves using formula, which if stated must be correct, by completing square (look for $(d-100)^2=10900 \Rightarrow d=..$) or even allow answers coming from a graphical calculator

A1: Awrt 204 m only

(b)

B1: States it is the initial height of the arrow above the ground. Do not allow " it is the height of the archer"

(c)

M1: Score for taking out a common factor of -0.002 from at least the d^2 and d terms

M1: For completing the square for their $\left(d^2-200d\right)$ term

A1: $=21.8-0.002(d-100)^2$ or exact equivalent

(d)

B1ft: For their '21.8+0.3' =22.1m

B1ft: For their 100m

Question	Scheme	Marks	AOs
12 (a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1	2.1
	$\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$	A1	1.1b
		(2)	
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient	M1	3.1b
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient	M1	1.1b
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1	3.1b
	Number of microbes ≈ 800	A1	1.1b
		(4)	
(c)	$N = 1000000 \Rightarrow \log_{10} N = 6$	M1	3.4
	We cannot 'extrapolate' the graph and assume that the model still holds	A1	3.5b
		(2)	
(d)	States that 'a' is the number of microbes 1 day after the start of the experiment	B1	3.2a
		(1)	
			(9 marks)

Question 12 continued	
Notes:	
(a)	
M1:	Takes logs of both sides and shows the addition law
M1:	Uses the power law, writes $\log_{10} N = \log_{10} a + b\log_{10} T$ and states $m = b$ **and** $c = \log_{10} a$
(b)	
M1:	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient. This would be implied by the sight of $b = 2.3$ **or** $a = 10^{1.8} \approx 63$
M1:	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient. This would be implied by the sight of $b = 2.3$ **and** $a = 10^{1.8} \approx 63$
M1:	Uses $T = 3 \Rightarrow N = aT^b$ with their a and b. This is implied by an attempt at $63 \times 3^{2.3}$
A1:	Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work. There is an alternative to this using a graphical approach.
M1:	Finds the value of $\log_{10} T$ from T =3. Accept as $T = 3 \Rightarrow \log_{10} T \approx 0.48$
M1:	Then using the line of best fit finds the value of $\log_{10} N$ from their "0.48" Accept $\log_{10} N \approx 2.9$
M1:	Finds the value of N from their value of $\log_{10} N$ $\log_{10} N \approx 2.9 \Rightarrow N = 10^{'2.9'}$
A1:	Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work
(c)	
M1	For using N = 1000000 and stating that $\log_{10} N = 6$
A1:	Statement to the effect that "we only have information for values of $\log N$ between 1.8 and 4.5 so we cannot be certain that the relationship still holds". "We cannot extrapolate with any certainty, we could only interpolate" There is an alternative approach that uses the formula.
M1:	Use N = 1000000 in their $N = 63 \times T^{2.3} \Rightarrow \log_{10} T = \dfrac{\log_{10}\left(\dfrac{1000000}{63}\right)}{2.3} \approx 1.83$.
A1:	The reason would be similar to the main scheme as we only have $\log_{10} T$ values from 0 to 1.2. We cannot 'extrapolate' the graph and assume that the model still holds
(d)	
B1:	Allow a numerical explanation $T = 1 \Rightarrow N = a1^b \Rightarrow N = a$ giving a is the value of N at T =1

Question	Scheme	Marks	AOs
13(a)	Attempts $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}$	M1	1.1b
	$\dfrac{dy}{dx}=\dfrac{\sqrt{3}\sin 2t}{\sin t}\quad\left(=2\sqrt{3}\cos t\right)$	A1	1.1b
		(2)	
(b)	Substitutes $t=\dfrac{2\pi}{3}$ in $\dfrac{dy}{dx}=\dfrac{\sqrt{3}\sin 2t}{\sin t}=\left(-\sqrt{3}\right)$	M1	2.1
	Uses gradient of normal = $-\dfrac{1}{dy/dx}=\left(\dfrac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P=\left(-1,-\dfrac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y+\dfrac{\sqrt{3}}{2}=\dfrac{1}{\sqrt{3}}(x+1)$	M1	2.1
	Completes proof $\Rightarrow 2x-2\sqrt{3}y-1=0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x=2\cos t$ and $y=\sqrt{3}\cos 2t$ into $2x-2\sqrt{3}y-1=0$	M1	3.1a
	Uses the identity $\cos 2t=2\cos^2 t-1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t-4\cos t-5=0$	A1	1.1b
	Finds $\cos t=\dfrac{5}{6},\cancel{-\dfrac{1}{2}}$	M1	2.4
	Substitutes their $\cos t=\dfrac{5}{6}$ into $x=2\cos t,\ \ y=\sqrt{3}\cos 2t$,	M1	1.1b
	$Q=\left(\dfrac{5}{3},\dfrac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
			(13 marks)

Question 13 continued

Notes:

(a)

M1: Attempts $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$ and achieves a form $k\frac{\sin 2t}{\sin t}$ Alternatively candidates may apply the double angle identity for $\cos 2t$ and achieve a form $k\frac{\sin t \cos t}{\sin t}$

A1: Scored for a correct answer, either $\frac{\sqrt{3}\sin 2t}{\sin t}$ or $2\sqrt{3}\cos t$

(b)

M1: For substituting $t = \frac{2\pi}{3}$ in their $\frac{\mathrm{d}y}{\mathrm{d}x}$ which must be in terms of t

M1: Uses the gradient of the normal is the negative reciprocal of the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$. This may be seen in the equation of l.

B1: States or uses (in their tangent or normal) that $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$

M1: Uses their numerical value of $-1/\frac{\mathrm{d}y}{\mathrm{d}x}$ with their $\left(-1, -\frac{\sqrt{3}}{2}\right)$ to form an equation of the normal at P

A1*: This is a proof and all aspects need to be correct. Correct answer only $2x - 2\sqrt{3}y - 1 = 0$

(c)

M1: For substituting $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$ to produce an equation in t. Alternatively candidates could use $\cos 2t = 2\cos^2 t - 1$ to set up an equation of the form $y = Ax^2 + B$.

M1: Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic equation in $\cos t$

In the alternative method it is for combining their $y = Ax^2 + B$ with $2x - 2\sqrt{3}y - 1 = 0$ to get an equation in just one variable

A1: For the correct quadratic equation $12\cos^2 t - 4\cos t - 5 = 0$

Alternatively the equations in x and y are $3x^2 - 2x - 5 = 0$ $12\sqrt{3}y^2 + 4y - 7\sqrt{3} = 0$

M1: Solves the quadratic equation in $\cos t$ (or x or y) and rejects the value corresponding to P.

M1: Substitutes their $\cos t = \frac{5}{6}$ or their $t = \arccos\left(\frac{5}{6}\right)$ in $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$

If a value of x or y has been found it is for finding the other coordinate.

A1: $Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$. Allow $x = \frac{5}{3}, y = \frac{7}{18}\sqrt{3}$ but do not allow decimal equivalents.

Question	Scheme	Marks	AOs
14(a)	Uses or implies $h = 0.5$	B1	1.1b
	For correct form of the trapezium rule =	M1	1.1b
	$\frac{0.5}{2}\{3+2.2958+2(2.3041+1.9242+1.9089)\}=4.393$	A1	1.1b
		(3)	
(b)	Any valid statement reason, for example • Increase the number of strips • Decrease the width of the strips • Use more trapezia	B1	2.4
		(1)	
(c)	For integration by parts on $\int x^2 \ln x \, dx$	M1	2.1
	$=\frac{x^3}{3}\ln x-\int\frac{x^2}{3}\,dx$	A1	1.1b
	$\int -2x+5\,dx=-x^2+5x \quad (+c)$	B1	1.1b
	All integration attempted and limits used Area of $S=\int_1^3 \frac{x^2\ln x}{3}-2x+5\,dx=\left[\frac{x^3}{9}\ln x-\frac{x^3}{27}-x^2+5x\right]_{x=1}^{x=3}$	M1	2.1
	Uses correct ln laws, simplifies and writes in required form	M1	2.1
	Area of $S=\frac{28}{27}+\ln 27 \quad (a=28, b=27, c=27)$	A1	1.1b
		(6)	
			(10 marks)

Question 14 continued
Notes:
(a) **B1:** States or uses the strip width $h = 0.5$. This can be implied by the sight of $\frac{0.5}{2}\{...\}$ in the trapezium rule **M1:** For the correct form of the bracket in the trapezium rule. Must be y values rather than x values $\{\text{first } y \text{ value} + \text{last } y \text{ value} + 2\times(\text{sum of other } y \text{ values})\}$ **A1:** 4.393 **(b)** **B1:** See scheme **(c)** **M1:** Uses integration by parts the right way around. Look for $\int x^2 \ln x \, dx = Ax^3 \ln x - \int Bx^2 \, dx$ **A1:** $\frac{x^3}{3}\ln x - \int \frac{x^2}{3} \, dx$ **B1:** Integrates the $-2x+5$ term correctly $= -x^2 + 5x$ **M1:** All integration completed and limits used **M1:** Simplifies using ln law(s) to a form $\frac{a}{b} + \ln c$ **A1:** Correct answer only $\frac{28}{27} + \ln 27$

Question	Scheme	Marks	AOs
15(a)	Attempts to differentiate using the quotient rule or otherwise	M1	2.1
	$f'(x) = \dfrac{e^{\sqrt{2}x-1} \times 8\cos 2x - 4\sin 2x \times \sqrt{2}e^{\sqrt{2}x-1}}{\left(e^{\sqrt{2}x-1}\right)^2}$	A1	1.1b
	Sets $f'(x) = 0$ and divides/ factorises out the $e^{\sqrt{2}x-1}$ terms	M1	2.1
	Proceeds via $\dfrac{\sin 2x}{\cos 2x} = \dfrac{8}{4\sqrt{2}}$ to $\Rightarrow \tan 2x = \sqrt{2}$ *	A1*	1.1b
		(4)	
(b)	(i) Solves $\tan 4x = \sqrt{2}$ and attempts to find the 2nd solution	M1	3.1a
	$x = 1.02$	A1	1.1b
	(ii) Solves $\tan 2x = \sqrt{2}$ and attempts to find the 1st solution	M1	3.1a
	$x = 0.478$	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)

M1: Attempts to differentiate by using the quotient rule with $u = 4\sin 2x$ and $v = e^{\sqrt{2}x-1}$ or alternatively uses the product rule with $u = 4\sin 2x$ and $v = e^{1-\sqrt{2}x}$

A1: For achieving a correct $f'(x)$. For the product rule $f'(x) = e^{1-\sqrt{2}x} \times 8\cos 2x + 4\sin 2x \times -\sqrt{2}e^{1-\sqrt{2}x}$

M1: This is scored for cancelling/ factorising out the exponential term. Look for an equation in just $\cos 2x$ and $\sin 2x$

A1*: Proceeds to $\tan 2x = \sqrt{2}$. This is a given answer.

(b) (i)

M1: Solves $\tan 4x = \sqrt{2}$ attempts to find the 2nd solution. Look for $x = \dfrac{\pi + \arctan\sqrt{2}}{4}$

Alternatively finds the 2nd solution of $\tan 2x = \sqrt{2}$ and attempts to divide by 2

A1: Allow awrt $x = 1.02$. The correct answer, with no incorrect working scores both marks

(b)(ii)

M1: Solves $\tan 2x = \sqrt{2}$ attempts to find the 1st solution. Look for $x = \dfrac{\arctan\sqrt{2}}{2}$

A1: Allow awrt $x = 0.478$. The correct answer, with no incorrect working scores both marks