8. (a) Given that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cos x) = -\sin x$$

show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that

$$x = \sec 2y$$

(b) find $\frac{dx}{dy}$ in terms of y.

(2)

(c) Hence find $\frac{dy}{dx}$ in terms of x.

(a) $e^{3x} + \ln 2x$, (3) (b) $(5 + x^2)^{\frac{3}{2}}$. (3)	Differentiate, with respect to x ,	
(3) (b) $(5+x^2)^{\frac{3}{2}}$.		
(b) $(5+x^2)^{\frac{3}{2}}$.	(a) $e^{3x} + \ln 2x$,	(3)
		(3)
	(b) $(5+x^2)^{\frac{1}{2}}$.	
		(3)

(Total 6 marks)

Q2

7. The curve *C* has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6\sin 2x + 4\cos 2x + 2}{\left(2 + \cos 2x\right)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$. Write your answer in the form y = ax + b, where a and b are exact constants.

4. (i) The curve *C* has equation

$$y = \frac{x}{9 + x^2}.$$

Use calculus to find the coordinates of the turning points of C.

(6)

(ii) Given that

$$y = (1 + e^{2x})^{\frac{3}{2}},$$

find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2} \ln 3$.

3. A curve C has equation

$$y = x^2 e^x.$$

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation.

(3)

(b) Hence find the coordinates of the turning points of C.

(3)

(c) Find $\frac{d^2y}{dx^2}$.

(2)

(d) Determine the nature of each turning point of the curve C.

(2)

Leave
blank

. Differentiate with respect to x , giving your answer in its simplest form,	
(a) $x^2 \ln(3x)$	(4)
(b) $\frac{\sin 4x}{x^3}$	(5)

2. A curve *C* has equation

$$y = e^{2x} \tan x$$
, $x \neq (2n+1)\frac{\pi}{2}$.

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where x = 0.

(2)

1.	The	curve	C	has	ec	uation
			_			0.000

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates (w, -32).

Find

(a) the value of w,

(2)

(b) the equation of the tangent to C at the point P in the form y = mx + c, where m and c are constants.





7.
$$f(x) = \frac{4x - 5}{(2x + 1)(x - 3)} - \frac{2x}{x^2 - 9}, \qquad x \neq \pm 3, \ x \neq -\frac{1}{2}$$

(a) Show that

$$f(x) = \frac{5}{(2x+1)(x+3)}$$

(5)

The curve C has equation y = f(x). The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(b) Find an equation of the normal to C at P.

(8)

7. A curve *C* has equation

$$y = 3\sin 2x + 4\cos 2x$$
, $-\pi \leqslant x \leqslant \pi$.

The point A(0, 4) lies on C.

(a) Find an equation of the normal to the curve C at A.

(5)

(b) Express y in the form $R\sin(2x+\alpha)$, where R > 0 and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve *C* with the *x*-axis. Give your answers to 2 decimal places.

- **6.** (a) Differentiate with respect to x,
 - (i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln (5x+2)$.

(3)

Given that $y = \frac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(5)

(b) show that $\frac{dy}{dx} = \frac{20}{(x+1)^3}$.

(5)

(c) Hence find $\frac{d^2y}{dx^2}$ and the real values of x for which $\frac{d^2y}{dx^2} = -\frac{15}{4}$.

(3)

	The point <i>P</i> lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The <i>x</i> -coordinate of <i>P</i> is Find an equation of the normal to the curve at the point <i>P</i> in the form $y = ax + b$, where $y = ax + b$, wh
	and b are constants.
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- (a) Differentiate with respect to x
 - (i) x^2e^{3x+2} ,

(4)

(4)

(b) Given that $x = 4 \sin(2y + 6)$, find $\frac{dy}{dx}$ in terms of x.

3.

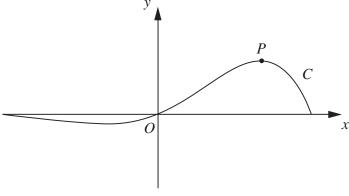


Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x , \quad -\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$$

(a) Find the *x* coordinate of the turning point *P* on *C*, for which x > 0 Give your answer as a multiple of π .

(6)

(b) Find an equation of the normal to C at the point where x = 0

(3)

1	$f(x) = 3e^x - \frac{1}{2} \ln x - 2, x > 0.$	
4.	$I(x) = 3e^{x} - \frac{1}{2} III x - 2, x > 0.$	

(a) Differentiate to find f'(x).

(3)

The curve with equation y = f(x) has a turning point at P. The x-coordinate of P is α .

(b) Show that $\alpha = \frac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \frac{1}{6} e^{-x_n}, \ x_0 = 1,$$

is used to find an approximate value for α .

(c) Calculate the values of x_1 , x_2 , x_3 and x_4 , giving your answers to 4 decimal places.

(2)

(d) By considering the change of sign of f'(x) in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)



1. (a) Find the value of $\frac{dy}{dx}$ at the point where x = 2 on the curve with equation

 $y = x^2 \sqrt{(5x - 1)}.$

(6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

3. The curve C has equation

$$x = 2 \sin y$$
.

(a) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C.

(1)

(b) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P.

(4)

(c) Find an equation of the normal to C at P. Give your answer in the form y = mx + c, where m and c are exact constants.



Leave
blank

1. Differentiate with respect to x	
(a) $\ln(x^2 + 3x + 5)$	(2)
(b) $\frac{\cos x}{x^2}$	(3)

7.	(a)	Differentiate	with	respect	to	x
----	-----	---------------	------	---------	----	---

- (i) $x^{\frac{1}{2}} \ln(3x)$
- (ii) $\frac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(b)	Given that	$x = 3\tan 2y$	find	$\frac{\mathrm{d}y}{\mathrm{d}x}$	in	terms	of x
-----	------------	----------------	------	-----------------------------------	----	-------	------

	(•

4. The point <i>P</i> is the point on the curve $x = 2 \tan \left(y + \frac{\pi}{12} \right)$ with <i>y</i> -coordinate $\frac{\pi}{4}$	- .
Find an equation of the normal to the curve at <i>P</i> .	
	(

Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.	
Give your answer in the form $y = ax + b$, where a and b are constants to be found.	(

	L
Sketch the graph of $y = \ln x $, stating the coordinates of any points of intersection with the axes.	
(3)	

5. Given that

$$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6}$$

(a) find
$$\frac{dx}{dy}$$
 in terms of y.

(2)

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(c)	Find an expression for	$\frac{\mathrm{d}^2 y}{\mathrm{d} x^2}$	in terms of x .	Give your	answer in it	s simplest form.
		$\mathrm{d}x^{-}$				

(4)

16

1.	The poir	nt <i>P</i> 1	ies on	the	curve	with	equation
1.	The pon	11 1	ics on	uic	curve	WILLI	equation

$$y=4e^{2x+1}.$$

The *y*-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P.

(2)

(b) Find the equation of the tangent to the curve at the point P in the form y = ax + b, where a and b are exact constants to be found.

Leave
blank

- **5.** (i) Differentiate with respect to x
 - (a) $y = x^3 \ln 2x$
 - (b) $y = (x + \sin 2x)^3$

(6)

Given that $x = \cot y$,

(ii) show that $\frac{dy}{dx} = \frac{-1}{1+x^2}$

8. (a) Express $2\cos 3x - 3\sin 3x$ in the form $R\cos (3x + \alpha)$, where R and α are constants, R > 0 and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures.

.

$$f(x) = e^{2x} \cos 3x$$

(b) Show that f'(x) can be written in the form

$$f'(x) = Re^{2x}\cos(3x + \alpha)$$

where R and α are the constants found in part (a).

(5)

(c) Hence, or otherwise, find the smallest positive value of x for which the curve with equation y = f(x) has a turning point.

(3)

2. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \quad x \neq \frac{5}{3}$$

The point P on C has x-coordinate 2. Find an equation of the normal to C at P in the form ax + by + c = 0, where a, b and c are integers.

	(7)

5.

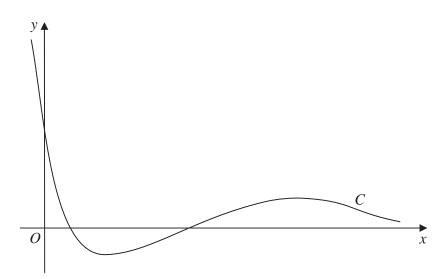


Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis.

(1)

Leave blank

(b) Show that C crosses the x-axis at x = 2 and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(c) Find
$$\frac{dy}{dx}$$
.

(d) Hence find the exact coordinates of the turning points of C.
