

A level Pure Maths: Practice Paper C mark scheme

1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States that: $A(x - 4)(3x + 1) + B(3x + 1) + C(x - 4)(x - 4) \equiv 18x^2 - 98x + 78$	M1	2.2a	7th Decompose algebraic fractions into partial fractions – repeated factors.
	Further states that: $A(3x^2 - 11x - 4) + B(3x + 1) + C(x^2 - 8x + 16) \equiv 18x^2 - 98x + 78$	M1	1.1b	
	Equates the various terms. Equating the coefficients of x^2 : $3A + C = 18$ Equating the coefficients of x : $-11A + 3B - 8C = -98$ Equating constant terms: $-4A + B + 16C = 78$	M1	2.2a	
	Makes an attempt to manipulate the expressions in order to find A , B and C . Obtaining two different equations in the same two variables would constitute an attempt.	M1	1.1b	
	Finds the correct value of any one variable: either $A = 4$, $B = -2$ or $C = 6$	A1	1.1b	
	Finds the correct value of all three variables: $A = 4$, $B = -2$, $C = 6$	A1	1.1b	
	(6 marks)			
Notes				
Alternative method				
Uses the substitution method, having first obtained this equation:				
$A(x - 4)(3x + 1) + B(3x + 1) + C(x - 4)(x - 4) \equiv 18x^2 - 98x + 78$				
Substitutes $x = 4$ to obtain $13B = -26$				
Substitutes $x = -\frac{1}{3}$ to obtain $\frac{169}{9}C = \frac{338}{3} \Rightarrow C = \frac{1014}{169} = 6$				
Equates the coefficients of x^2 : $3A + C = 18$				
Substitutes the found value of C to obtain $3A = 12$				

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2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Differentiates 4^x to obtain $4^x \ln 4$	M1	1.1b	7th Differentiate simple functions defined implicitly.
	Differentiates $2xy$ to obtain $2x \frac{dy}{dx} + 2y$	M1	2.2a	
	Rearranges $4^x \ln 4 = 2x \frac{dy}{dx} + 2y$ to obtain $\frac{dy}{dx} = \frac{4^x \ln 4 - 2y}{2x}$	A1	1.1b	
	Makes an attempt to substitute (2, 4)	M1	1.1b	
	States fully correct final answer: $4 \ln 4 - 2$ Accept $\ln 256 - 2$	A1	1.1b	
(5 marks)				
Notes				

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3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Correctly states $\cos(5x + 2x) \equiv \cos 5x \cos 2x - \sin 5x \sin 2x$	M1	1.1b	6th Integrate using trigonometric identities.
	Correctly states $\cos(5x - 2x) \equiv \cos 5x \cos(-2x) - \sin 5x \sin(-2x)$ or states $\cos(5x - 2x) \equiv \cos 5x \cos(2x) + \sin 5x \sin(2x)$	M1	1.1b	
	Adds the two above expressions and states $\cos 7x + \cos 3x \equiv 2 \cos 5x \cos 2x$	A1	1.1b	
		(3)		
	(b)	States that $\int (\cos 5x \cos 2x) dx = \frac{1}{2} \int (\cos 7x + \cos 3x) dx$	M1	2.2a
Makes an attempt to integrate. Changing cos to sin constitutes an attempt.		M1	1.1b	
Correctly states the final answer $\frac{1}{14} \sin 7x + \frac{1}{6} \sin 3x + C$ o.e.		A1	1.1b	
		(3)		
(6 marks)				
Notes				
(b) Student does not need to state ‘+C’ to be awarded the first method mark. Must be stated in the final answer.				

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4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to substitute $t = 0$ into $T(t) = T_R + (90 - T_R)e^{-\frac{1}{20}t}$. For example, $T(t) = T_R + (90 - T_R)e^0$ or $T(t) = T_R + (90 - T_R)$ is seen.	M1	3.1a	6th Set up and use exponential models of growth and decay.
	Concludes that the T_R terms will always cancel at $t = 0$, therefore the room temperature does not influence the initial coffee temperature.	B1	3.5a	
		(2)		
(b)	Makes an attempt to substitute $T_R = 20$ and $t = 10$ into $T(t) = T_R + (90 - T_R)e^{-\frac{1}{20}t}$. For example, $T(10) = 20 + (90 - 20)e^{-\frac{1}{20}(10)}$ is seen.	M1	1.1b	6th Set up and use exponential models of growth and decay.
	Finds $T(10) = 62.457...^{\circ}\text{C}$. Accept awrt 62.5° .	A1	1.1b	
		(2)		
(4 marks)				
Notes				

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5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. ‘Assumption: there exists a number n such that n is odd and $n^3 + 1$ is also odd.’	B1	3.1	7th Complete proofs using proof by contradiction.
	Defines an odd number. ‘Let $2k + 1$ be an odd number.’	B1	2.2a	
	Successfully calculates $(2k + 1)^3 + 1$ $(2k + 1)^3 + 1 \circ (8k^3 + 12k^2 + 6k + 1) + 1 \circ 8k^3 + 12k^2 + 6k + 2$	M1	1.1b	
	Factors the expression and concludes that this number must be even. $8k^3 + 12k^2 + 6k + 2 \circ 2(4k^3 + 6k^2 + 3k + 1)$ $2(4k^3 + 6k^2 + 3k + 1)$ is even.	M1	1.1b	
	Makes a valid conclusion. This contradicts the assumption that there exists a number n such that n is odd and $n^3 + 1$ is also odd, so if n is odd, then $n^3 + 1$ is even.	B1	2.4	
(5 marks)				
Notes				
Alternative method				
Assume the opposite is true: there exists a number n such that n is odd and $n^3 + 1$ is also odd. (B1)				
If $n^3 + 1$ is odd, then n^3 is even. (B1)				
So 2 is a factor of n^3 . (M1)				
This implies 2 is a factor of n . (M1)				
This contradicts the statement n is odd. (B1)				

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6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Recognises that the identity $\sin^2 t + \cos^2 t \equiv 1$ can be used to find the cartesian equation.	M1	2.2a	6th Convert between parametric equations and cartesian forms using trigonometry.
	States $\sin t = \frac{y}{2}$ or $\sin^2 t = \frac{y^2}{4}$ Also states $\cos^2 t = \frac{1}{x-1}$	M1	1.1b	
	Substitutes $\sin^2 t = \frac{y^2}{4}$ and $\cos^2 t = \frac{1}{x-1}$ into $\sin^2 t + \cos^2 t \equiv 1$ $\frac{y^2}{4} + \frac{1}{x-1} = 1 \Rightarrow \frac{y^2}{4} = \frac{x-2}{x-1}$	M1	1.1b	
	Solves to find $y = \sqrt{\frac{4x-8}{x-1}}$, accept $y = \sqrt{\frac{8-4x}{1-x}}$, $x < 1$ or $x \dots 2$	A1	1.1b	
(4 marks)				
Notes				

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7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Understands that for the series to be convergent $ r < 1$ or states $ -4x < 1$	M1	2.2a	6th Understand convergent geometric series and the sum to infinity.
	Correctly concludes that $ x < \frac{1}{4}$. Accept $-\frac{1}{4} < x < \frac{1}{4}$	A1	1.1b	
		(2)		
(b)	Understands to use the sum to infinity formula. For example, states $\frac{1}{1+4x} = 4$	M1	2.2a	5th Understand sigma notation.
	Makes an attempt to solve for x. For example, $4x = -\frac{3}{4}$ is seen.	M1	1.1b	
	States $x = -\frac{3}{16}$	A1	1.1b	
		(3)		
(5 marks)				
Notes				

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8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Finds $f(1.9) = -0.2188\dots$ and $f(2.0) = (+)0.1606\dots$	M1	1.1b	5th
	Change of sign and continuous function in the interval $[1.9, 2.0] \Rightarrow$ root	A1	2.4	Use a change of sign to locate roots.
		(2)		
(b)	Makes an attempt to differentiate $f(x)$	M1	2.2a	6th
	Correctly finds $f'(x) = -9\sin^2 x \cos x + \sin x$	A1	1.1b	Solve equations approximately using the Newton-Raphson method.
	Finds $f(1.95) = -0.0348\dots$ and $f'(1.95) = 3.8040\dots$	M1	1.1b	
	Attempts to find x_1 $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = 1.95 - \frac{-0.0348\dots}{3.8040\dots}$	M1	1.1b	
	Finds $x_1 = 1.959$	A1	1.1b	
		(5)		
(7 marks)				
Notes				
(a) Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.				

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9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States $-a + b = 10$ and $7a + 5b = 2$	M1	2.2a	6th Solve geometric problems using vectors in 3 dimensions
	Makes an attempt to solve the pair of simultaneous equations. Attempt could include making a substitution or multiplying the first equation by 5 or by 7.	M1	1.1b	
	Finds $a = -4$	A1	1.1b	
	Find $b = 6$	A1	1.1b	
	States $-2abc = -96$	M1	2.2a	
	Finds $c = -2$	A1	1.1b	
				(6 marks)
Notes				

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10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. ‘Assumption: there exist positive integer solutions to the statement $x^2 - y^2 = 1$ ’	B1	3.1	7th Complete proofs using proof by contradiction.
	Sets up the proof by factorising $x^2 - y^2$ and stating $(x - y)(x + y) = 1$	M1	2.2a	
	States that there is only one way to multiply to make 1: $1 \times 1 = 1$ and concludes this means that: $x - y = 1$ $x + y = 1$	M1	1.1b	
	Solves this pair of simultaneous equations to find the values of x and y : $x = 1$ and $y = 0$	M1	1.1b	
	Makes a valid conclusion. $x = 1, y = 0$ are not both positive integers, which is a contradiction to the opening statement. Therefore there do not exist positive integers x and y such that $x^2 - y^2 = 1$	B1	2.4	
(5 marks)				
Notes				

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11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Understands the need to complete the square, and makes an attempt to do this. For example, $(x-4)^2$ is seen.	M1	2.2a	6th Find the domain and range of inverse functions.
	Correctly writes $g(x)=(x-4)^2-9$	A1	1.1b	
	Demonstrates an understanding of the method for finding the inverse is to switch the x and y . For example, $x=(y-4)^2-9$ is seen.	B1	2.2a	
	Makes an attempt to rearrange to make y the subject. Attempt must include taking the square root.	M1	1.1b	
	Correctly states $g^{-1}(x)=\sqrt{x+9}+4$	A1	1.1b	
	Correctly states domain is $x > -9$ and range is $y > 4$	B1	3.2b	
(6 marks)				
Notes				

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12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States that: $A(4 - x)(x + 5) + B(x - 3)(x + 5) + C(x - 3)(4 - x) \equiv 4x^2 + x - 23$	M1	2.2a	6th Decompose algebraic fractions into partial fractions – three linear factors.
	Further states that: $A(-x^2 - x + 20) + B(x^2 + 2x - 15) + C(-x^2 + 7x - 12) \equiv 4x^2 + x - 23$	M1	1.1b	
	Equates the various terms. Equating the coefficients of x^2 : $-A + B - C = 4$ Equating the coefficients of x : $-A + 2B + 7C = 1$ Equating constant terms: $20A - 15B - 12C = -23$	M1*	2.2a	
	Makes an attempt to manipulate the expressions in order to find A , B and C . Obtaining two different equations in the same two variables would constitute an attempt.	M1*	1.1b	
	Finds the correct value of any one variable: either $A = 2$, $B = 5$ or $C = -1$	A1*	1.1b	
	Finds the correct value of all three variables: $A = 2$, $B = 5$, $C = -1$	A1	1.1b	
	(6 marks)			
Notes				
Alternative method				
Uses the substitution method, having first obtained this equation: $A(4 - x)(x + 5) + B(x - 3)(x + 5) + C(x - 3)(4 - x) \equiv 4x^2 + x - 23$				
Substitutes $x = 4$ to obtain $9B = 45$ (M1)				
Substitutes $x = 3$ to obtain $8A = 16$ (M1)				
Substitutes $x = -5$ to obtain $-72C = 72$ (A1)				

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13	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Finds $\frac{dy}{dx} = 3x^2 + 12x - 12$	M1	1.1b	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
	Finds $\frac{d^2y}{dx^2} = 6x + 12$	M1	1.1b	
	States that $\frac{d^2y}{dx^2} = 6x + 12 \leq 0$ for all $-5 \leq x \leq -3$ and concludes this implies C is concave over the given interval.	B1	3.2a	
		(3)		
(b)	States or implies that a point of inflection occurs when $\frac{d^2y}{dx^2} = 0$	M1	3.1a	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
	Finds $x = -2$	A1	1.1b	
	Substitutes $x = -2$ into $y = x^3 + 6x^2 - 12x + 6$, obtaining $y = 46$	A1	1.1b	
		(3)		
(6 marks)				
Notes				

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14	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to find $\int \sin 4x(1 - \cos 4x)^3 \, dx$. Raising the power by 1 would constitute an attempt.	M1	2.2a	6th Integrate using the reverse chain rule.
	States a fully correct answer $\int \sin 4x(1 - \cos 4x)^3 \, dx = \frac{1}{16}(1 - \cos 4x)^4 + C$	M1	2.2a	
	Makes an attempt to substitute the limits $\frac{1}{16} \left[(1 - 0)^4 - \left(1 - \frac{1}{2}\right)^4 \right]$	M1 ft	1.1b	
	Correctly states answer is $\frac{15}{256}$	A1 ft	1.1b	
(4 marks)				
Notes				
Student does not need to state ‘+C’ to be awarded the second method mark.				
Award ft marks for a correct answer using an incorrect initial answer.				

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15	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to set up a long division. For example, $2x^2 - x - 1 \overline{) 4x^2 - 4x - 9}$ is seen.	M1	2.2a	7th Expand rational functions using partial fraction decomposition.
	Long division completed so that a 2 is seen in the quotient and a remainder of $-2x - 7$ is also seen. $\begin{array}{r} 2 \\ 2x^2 - x - 1 \overline{) 4x^2 - 4x - 9} \\ \underline{4x^2 - 2x - 2} \\ -2x - 7 \end{array}$	M1	1.1b	
	States $B(x - 1) + C(2x + 1)^0 = 2x - 7$	M1	2.2a	
	Either equates variables or makes a substitution in an effort to find B or C .	M1	2.2a	
	Finds $B = 4$	A1	1.1b	
	Finds $C = -3$	A1	1.1b	
		(6)		
(b)	Correctly writes $4(2x + 1)^{-1}$ or $4(1 + 2x)^{-1}$ as $4 \left(1 + (-1)(2x) + \frac{(-1)(-2)(2)^2 x^2}{2} + \dots \right)$	M1 ft	2.2a	6th Understand the binomial theorem for rational n .
	Simplifies to obtain $4 - 8x + 16x^2 + \dots$	A1 ft	1.1b	
	Correctly writes $\frac{-3}{x-1}$ as $\frac{3}{1-x}$	M1 ft	2.2a	
	Correctly writes $3(1 - x)^{-1}$ as $3 \left(1 + (-1)(-x) + \frac{(-1)(-2)(-1)^2 (-x)^2}{2} + \dots \right)$	M1 ft	2.2a	
	Simplifies to obtain $3 + 3x + 3x^2 + \dots$	A1 ft	1.1b	
	States the correct final answer: $9 - 5x + 19x^2$	A1 ft	1.1b	
		(6)		

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(c)	The expansion is only valid for $ x < \frac{1}{2}$	B1	3.2b	6th Understand the conditions for validity of the binomial theorem for rational n.
		(1)		
(13 marks)				
Notes (a) Alternative method. Writes the RHS as a single fraction. Obtains $4x^2 - 4x - 9 = A(2x + 1)(x - 1) + B(x - 1) + C(2x + 1)$ Substitutes $x = 1$ to obtain $C = -3$ Substitutes $x = -\frac{1}{2}$ to obtain $B = 4$ Compares coefficients of x^2 to obtain $A = 2$ (b) Award all 6 marks for a correct answer using their incorrect values of A, B and/or C from part a.				

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16	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$	M1	3.1b	8th Solve differential equations in a range of contexts
	Deduces that $V = \pi r^2 h = 1600\pi h$	M1	3.1b	
	Finds $\frac{dV}{dh} = 1600\pi$ and/or $\frac{dh}{dV} = \frac{1}{1600\pi}$	M1	1.1b	
	States $\frac{dV}{dt} = 4000\pi - 50\pi h$	M1	3.1b	
	Makes an attempt to find $\frac{dh}{dt} = (4000\pi - 50\pi h) \times \frac{1}{1600\pi}$	M1	1.1b	
	Shows a clear logical progression to state $160 \frac{dh}{dt} = 400 - 5h$	A1	1.1b	
		(6)		
(b)	Separates the variables $\int \left(\frac{1}{400 - 5h} \right) dh = \int \frac{1}{160} dt$	M1	2.2a	8th Solve differential equations in a range of contexts.
	Finds $-\frac{1}{5} \ln(400 - 5h) = \frac{t}{160} + C$	A1	1.1b	
	Uses the fact that $t = 0$ when $h = 50$ m to find C $C = -\frac{1}{5} \ln(150)$	M1	1.1b	
	Substitutes $h = 60$ into the equation $-\frac{1}{5} \ln(400 - 300) = \frac{t}{160} - \frac{1}{5} \ln(150)$	M1	3.1b	
	Uses law of logarithms to write $\frac{1}{5} \ln(150) - \frac{1}{5} \ln(100) = \frac{t}{160}$ $\Rightarrow \frac{1}{5} \ln\left(\frac{150}{100}\right) = \frac{t}{160}$	M1	2.2a	
	States correct final answer $t = 32 \ln\left(\frac{3}{2}\right)$ minutes.	A1	1.1b	
		(6)		
(12 marks)				
Notes				