1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to substitute $k = 1$, $k = 2$ and $k = 4$ into $a_k = 2^k + 1, k \geqslant 1$	M1	1.1b	5th Understand disproof by counter example.
	Shows that $a_1 = 3$, $a_2 = 5$ and $a_4 = 17$ and these are prime numbers.	A1	1.1b	
		(2)		
(b)	Substitutes a value of k that does not yield a prime number. For example, $a_3 = 9$ or $a_5 = 33$	A1	1.1b	5th Understand disproof by counter example.
	Concludes that their number is not prime. For example, states that 9 = 3 × 3, so 9 is not prime.	B1	2.4	
		(2)		
				(4 marks)

Notes

2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Finds $\lvert a \rvert = \sqrt{(4)^2 + (-1)^2 + (3)^2} = \sqrt{26}$	**M1**	1.1b	5th Find the magnitude of a vector in 3 dimensions.
	States $\cos\theta_y = -\frac{1}{\sqrt{26}}$	**M1**	1.1b	
	Solves to find $\theta_y = 101.309...°$. Accept awrt 101.3°	**A1**	1.1b	
		(3)		
				(3 marks)

Notes

3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Deduces from $3\sin\left(\frac{x}{6}\right)^3 - \frac{1}{10}x - 1 = 0$ that $3\sin\left(\frac{x}{6}\right)^3 = \frac{1}{10}x + 1$	M1	1.1b	5th Understand the concept of roots of equations.
	States $\left(\frac{x}{6}\right)^3 = \arcsin\left(\frac{1}{3} + \frac{1}{30}x\right)$	M1	1.1b	
	Multiplies by 6^3 and then takes the cube root: $x = 6\left(\sqrt[3]{\arcsin\left(\frac{1}{3} + \frac{1}{30}x\right)}\right)$	A1	1.1b	
		(3)		
(b)	Attempts to use iterative procedure to find subsequent values.	M1	1.1b	6th Solve equations approximately using the method of iteration.
	Correctly finds: $x_1 = 4.716$ $x_2 = 4.802$ $x_3 = 4.812$ $x_4 = 4.814$	A1	1.1b	
		(2)		
				(5 marks)

Notes

(b) Award M1 if finds at least one correct answer.

4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Recognises that two subsequent values will divide to give an equal ratio and sets up an appropriate equation. $\frac{2k^2}{4k}=\frac{4k}{k+2}$	**M1**	2.2a	4th Understand simple geometric sequences.
	Makes an attempt to solve the equation. For example, $2k^3+4k^2=16k^2$ or $2k^3-12k^2=0$	**M1**	1.1b	
	Factorises to get $2k^2(k-6)=0$	**M1**	1.1b	
	States the correct solution: k = 6. $k \neq 0$ or k = 0 is trivial may also be seen, but is not required.	**A1**	1.1b	
				(4 marks)

Notes

5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to set up a long division. For example: $x^2 - 2x - 15\overline{)x^4 + 2x^3 - 29x^2 - 48x + 90}$ is seen.	**M1**	2.2a	6th Decompose algebraic fractions into partial fractions – three linear factors.
	Award 1 accuracy mark for each of the following: x^2 seen, $4x$ seen, -6 seen. $x^2 + 4x - 6$ $x^2 - 2x - 15\overline{)x^4 + 2x^3 - 29x^2 - 47x + 77}$ $\underline{x^4 - 2x^3 - 15x^2}$ $4x^3 - 14x^2 - 47x$ $\underline{4x^3 - 8x^2 - 60x}$ $-6x^2 + 13x + 77$ $\underline{-6x^2 + 12x + 90}$ $x - 13$	**A3**	1.1b	
	Equates the various terms to obtain the equation: $x - 13 = V(x - 5) + W(x + 3)$ Equating the coefficients of x: $V + W = 1$ Equating constant terms: $-5V + 3W = -13$	**M1**	2.2a	
	Multiplies one or or both of the equations in an effort to equate one of the two variables.	**M1**	1.1b	
	Finds $W = -1$ and $V = 2$.	**A1**	1.1b	
				(7 marks)

Notes

6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Use Pythagoras' theorem to show that the length of $OB = 2\sqrt{3}$ or $OD = 2\sqrt{3}$ or states $BD = 4\sqrt{3}$	**M1**	2.2a	6th Solve problems involving arc length and sector area in context.
	Makes an attempt to find ÐDAB or ÐDCB. For example, $\cos\angle DAO = \frac{2}{4}$ is seen.	**M1**	2.2a	
	Correctly states that $\angle DAB = \frac{2\pi}{3}$ or $\angle DCB = \frac{2\pi}{3}$	**A1**	1.1b	
	Makes an attempt to find the area of the sector with a radius of 4 and a subtended angle of $\frac{2\pi}{3}$ For example, $A = \frac{1}{2} \times 4^2 \times \frac{2\pi}{3}$ is shown.	**M1**	2.2a	
	Correctly states that the area of the sector is $\frac{16\pi}{3}$	**A1**	1.1b	
	Recognises the need to subtract the sector area from the area of the rhombus in an attempt to find the shaded area. For example, $\frac{16\pi}{3} - 8\sqrt{3}$ is seen.	**M1**	3.2a	
	Recognises that to find the total shaded area this number will need to be multiplied by 2. For example, $2 \times \left(\frac{16\pi}{3} - 8\sqrt{3}\right)$	**M1**	3.2a	
	Using clear algebra, correctly manipulates the expression and gives a clear final answer of $\frac{2}{3}\left(16\pi - 24\sqrt{3}\right)$	**A1**	1.1b	
				(8 marks)

Notes

7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to rearrange $x=\frac{1+4t}{1-t}$ to make t the subject. For example, $x-xt=1+4t$ is seen.	M1	2.2a	5th Convert between parametric equations and cartesian forms using substitution.
	Correctly states $t=\frac{x-1}{4+x}$	A1	1.1b	
	Makes an attempt to substitute $t=\frac{x-1}{4+x}$ into $y=\frac{2+bt}{1-t}$ For example, $y=\frac{2+\frac{bx-b}{x+4}}{1-\frac{x-1}{x+4}}=\frac{\frac{2x+8+bx-b}{x+4}}{\frac{x+4-x+1}{x+4}}$ is seen.	M1	2.2a	
	Simplifies the expression showing all steps. For example, $y=\frac{2x+8+bx-b}{5}=\left(\frac{2+b}{5}\right)x+\left(\frac{8-b}{5}\right)$	A1	1.1b	
		(4)		
(b)	Interprets the gradient of line being -1 as $\frac{2+b}{5}=-1$ and finds $b=-7$	M1	2.2a	5th Convert between parametric equations and cartesian forms using substitution.
	Substitutes $t=-1$ to find $x=-\frac{3}{2}$ and $y=\frac{9}{2}$ And substitutes t = 0 to find x = 1 and y = 2	M1	1.1b	
	Makes an attempt to use Pythagoras' Theorem to find the length of the line: $\sqrt{\left(\frac{5}{2}\right)^2+\left(\frac{5}{2}\right)^2}$	M1	1.1b	
	Correctly finds the length of the line segment, $\frac{5\sqrt{2}}{2}$ or states $a=\frac{5}{2}$	A1	1.1b	
		(4)		
				(8 marks)

Notes

8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Differentiates $u = 4t^{\frac{2}{3}}$ obtaining $\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{8}{3}t^{-\frac{1}{3}}$ and differentiates $v = t^2 + 1$ obtaining $\frac{\mathrm{d}v}{\mathrm{d}t} = 2t$	**M1**	1.1b	6th Differentiate using the product rule.
	Makes an attempt to substitute the above values into the product rule formula: $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}t} - u\frac{\mathrm{d}v}{\mathrm{d}t}}{v^2}$	**M1**	2.2a	
	Finds $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\frac{8}{3}t^{\frac{5}{3}} + \frac{8}{3}t^{-\frac{1}{3}} - 8t^{\frac{5}{3}}}{\left(t^2+1\right)^2}$	**M1**	1.1b	
	Fully simplfies using correct algebra to obtain $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{8\left(1-2t^2\right)}{3\sqrt[3]{t}\left(t^2+1\right)^2}$	**A1**	2.4	
		(4)		
(b)	Makes an attempt to substitute $t = 2$ into $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{8\left(1-2t^2\right)}{3\sqrt[3]{t}\left(t^2+1\right)^2} = \frac{8\left(1-2(2)^2\right)}{3\sqrt[3]{2}\left(2^2+1\right)^2}$	**M1 ft**	1.1b	6th Differentiate using the product rule.
	Correctly finds $\frac{\mathrm{d}H}{\mathrm{d}t} = -0.592...$ and concludes that as $\frac{\mathrm{d}H}{\mathrm{d}t} < 0$ the toy soldier was decreasing in height after 2 seconds.	**B1 ft***	3.5a	
		(2)		

(c)	$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{8\left(1-2t^2\right)}{3\sqrt[3]{t}\left(t^2+1\right)^2} = 0$ or $8-16t^2=0$ at a turning point.	**M1 ft**	1.1b	6th Differentiate using the product rule.
	Solves $8-16t^2=0$ to find $t=\frac{1}{\sqrt{2}}$ Can also state $t \neq -\frac{1}{\sqrt{2}}$	**A1 ft**	1.1b	
		(2)		
(8 marks)				

Notes

(b) Award ft marks for a correct answer using an incorrect answer from part **a**.

B1: Can also state $\frac{\mathrm{d}H}{\mathrm{d}t} < 0$ as the numerator of $\frac{\mathrm{d}H}{\mathrm{d}t}$ is negative and the denominator is positive.

Award ft marks for a correct answer using an incorrect answer from part **a**.

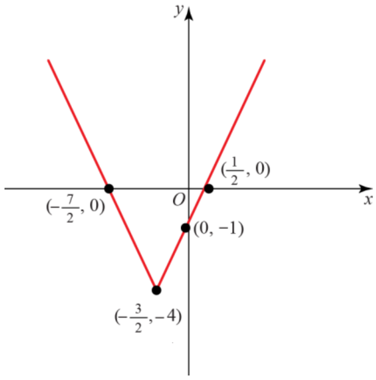
9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Recognises the need to write $\tan^4 x \equiv \tan^2 x \tan^2 x$	M1	2.2a	6th Integrate using trigonometric identities.
	Recognises the need to write $\tan^2 x \tan^2 x \equiv \left(\sec^2 x - 1\right)\tan^2 x$	M1	2.2a	
	Multiplies out the bracket and makes a further substitution $\left(\sec^2 x - 1\right)\tan^2 x$ $\equiv \sec^2 x \tan^2 x - \tan^2 x$ $\equiv \sec^2 x \tan^2 x - \left(\sec^2 x - 1\right)$	M1	2.2a	
	States the fully correct final answer $\sec^2 x \tan^2 x + 1 - \sec^2 x$	A1	1.1b	
		(4)		
(b)	States or implies that $\int \sec^2 x \, dx = \tan x$	M1	1.1b	6th Integrate using the reverse chain rule.
	States fully correct integral $\int \tan^4 x \, dx = \frac{1}{3}\tan^3 x + x - \tan x + C$	M1	2.2a	
	Makes an attempt to substitute the limits. For example, $\left[\frac{1}{3}\tan^3 x + x - \tan x\right]_0^{\frac{\pi}{4}} = \left(\frac{1}{3}\left(\tan\frac{\pi}{4}\right)^3 + \frac{\pi}{4} - \tan\frac{\pi}{4}\right) - (0)$ is seen.	M1 ft	1.1b	
	Begins to simplify the expression $\frac{1}{3} + \frac{\pi}{4} - 1$	M1 ft	1.1b	
	States the correct final answer $\frac{3\pi - 8}{12}$	A1 ft	1.1b	
		(5)		
				(9 marks)

Notes

(b) Student does not need to state '+C' to be awarded the second method mark.

(b) Award ft marks for a correct answer using an incorrect initial answer.

10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. ‘Assumption: given a rational number a and an irrational number b, assume that $a - b$ is rational.’	**B1**	3.1	7th Complete proofs using proof by contradiction.
	Sets up the proof by defining the different rational and irrational numbers. The choice of variables does not matter. Let $a = \frac{m}{n}$ As we are assuming $a - b$ is rational, let $a - b = \frac{p}{q}$ So $a - b = \frac{p}{q} \Rightarrow \frac{m}{n} - b = \frac{p}{q}$	**M1**	2.2a	
	Solves $\frac{m}{n} - b = \frac{p}{q}$ to make b the subject and rewrites the resulting expression as a single fraction: $\frac{m}{n} - b = \frac{p}{q} \Rightarrow b = \frac{m}{n} - \frac{p}{q} = \frac{mq - pn}{nq}$	**M1**	1.1b	
	Makes a valid conclusion. $b = \frac{mq - pn}{nq}$, which is rational, contradicts the assumption b is an irrational number. Therefore the difference of a rational number and an irrational number is irrational.	**B1**	2.4	
				(4 marks)
	Notes			

11	Scheme		Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	**Figure 1** 	Graph has a distinct V-shape.	**M1**	2.2a	5th Sketch the graph of the modulus function of a linear function.
		Labels vertex $\left(-\frac{3}{2}, -4\right)$	**A1**	2.2a	
		Finds intercept with the y-axis.	**M1**	1.1b	
		Makes attempt to find x-intercept, for example states that $\lvert 2x+3 \rvert - 4 = 0$	**M1**	2.2a	
		Successfully finds both x-intercepts.	**A1**	1.1b	
			(5)		
(b)	Recognises that there are two solutions. For example, writing $2x+3=-\frac{1}{4}x+2$ and $-(2x+3)=-\frac{1}{4}x+2$		**M1**	2.2a	5th Solve equations involving the modulus function.
	Makes an attempt to solve both questions for x, by manipulating the algebra.		**M1**	1.1b	
	Correctly states $x = -\frac{4}{9}$ or $x = -\frac{20}{7}$. Must state both answers.		**A1**	1.1b	
	Makes an attempt to substitute to find y.		**M1**	1.1b	
	Correctly finds y and states both sets of coordinates correctly $\left(-\frac{4}{9}, -\frac{17}{9}\right)$ and $\left(-\frac{20}{7}, -\frac{9}{7}\right)$		**A1**	1.1b	
			(5)		
					(10 marks)

Notes

12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Writes $(\sin 3\theta + \cos 3\theta)^2 \equiv (\sin 3\theta + \cos 3\theta)(\sin 3\theta + \cos 3\theta)$ $\equiv \sin^2 3\theta + 2\sin 3\theta \cos 3\theta + \cos^2 3\theta$	**M1**	1.1b	7th Use addition formulae and/or double-angle formulae to solve equations.
	Uses $\sin^2 3\theta + \cos^2 3\theta \equiv 1$ and $2\sin 3\theta \cos 3\theta \equiv \sin 6\theta$ to write: $(\sin 3\theta + \cos 3\theta)^2 \equiv 1 + \sin 6\theta$ Award one mark for each correct use of a trigonometric identity.	**A2**	2.2a	
		(3)		
(b)	States that: $1 + \sin 6\theta = \dfrac{2+\sqrt{2}}{2}$	**B1**	2.2a	7th Use addition formulae and/or double-angle formulae to solve equations.
	Simplifies this to write: $\sin 6\theta = \dfrac{\sqrt{2}}{2}$	**M1**	1.1b	
	Correctly finds $6\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{9\pi}{4}, \dfrac{11\pi}{4}$ Additional answers might be seen, but not necessary in order to award the mark.	**M1**	1.1b	
	States $\theta = \dfrac{\pi}{24}, \dfrac{3\pi}{24}$ Note that $\theta \neq \dfrac{9\pi}{24}, \dfrac{11\pi}{24}$. For these values 3θ lies in the third quadrant, therefore $\sin 3\theta$ and $\cos 3\theta$ are both negative and cannot be equal to a positive surd.	**A1**	1.1b	
		(4)		
				(7 marks)

Notes

6b

Award all 4 marks if correct final answer is seen, even if some of the 6θ angles are missing in the preceding step.

13	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Correctly writes $6(2+3x)^{-1}$ as: $6\left(2^{-1}\left(1+\frac{3}{2}x\right)^{-1}\right)$ or $3\left(1+\frac{3}{2}x\right)^{-1}$	M1	2.2a	6th Understand the binomial theorem for rational n.
	Completes the binomial expansion: $3\left(1+\frac{3}{2}x\right)^{-1}=3\left(1+(-1)\left(\frac{3}{2}\right)x+\frac{(-1)(-2)\left(\frac{3}{2}\right)^2x^2}{2}+...\right)$	M1	2.2a	
	Simplifies to obtain $3-\frac{9}{2}x+\frac{27}{4}x^2+...$	A1	1.1b	
	Correctly writes $4(3-5x)^{-1}$ as: $4\left(3^{-1}\left(1-\frac{5}{3}x\right)^{-1}\right)$ or $\frac{4}{3}\left(1-\frac{5}{3}x\right)^{-1}$	M1	2.2a	
	Completes the binomial expansion: $\frac{4}{3}\left(1-\frac{5}{3}x\right)^{-1}=\frac{4}{3}\left(1+(-1)\left(-\frac{5}{3}\right)x+\frac{(-1)(-2)\left(-\frac{5}{3}\right)^2x^2}{2}+...\right)$	M1	2.2a	
	Simplifies to obtain $\frac{4}{3}+\frac{20}{9}x+\frac{100}{27}x^2+...$	A1	1.1b	
	Simplifies by subtracting to obtain $\frac{5}{3}-\frac{121}{18}x+\frac{329}{108}x^2+...$ Reference to the need to subtract, or the subtracting shown, must be seen in order to award the mark.	A1	1.1b	
		(7)		

(b)	Makes an attempt to substitute $x = 0.01$ into f(x). For example, $\frac{6}{2+3(0.01)} - \frac{4}{3-5(0.01)}$ is seen.	**M1**	1.1b	6th Understand the binomial theorem for rational n.
	States the answer 1.5997328	**A1**	1.1b	
		(2)		
(c)	Makes an attempt to substitute $x = 0.01$ into $\frac{5}{3}-\frac{121}{18}x-\frac{329}{108}x^2+...$ For example $\frac{5}{3}-\frac{121}{18}(0.01)+\frac{329}{108}(0.01)^2+...$ is seen.	**M1 ft**	1.1b	6th Understand the binomial theorem for rational n.
	States the answer 1.59974907… Accept awrt 1.60.	**M1 ft**	1.1b	
	Finds the percentage error: 0.0010%	**A1 ft**	1.1b	
		(3)		
				(12 marks)

Notes

(a) If one expansion is correct and one is incorrect, or both are incorrect, award the final accuracy mark if they are subtracted correctly.

(c) Award all 3 marks for a correct answer using their incorrect answer from part (a).

14	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Uses $a_n = a + (n-1)d$ substituting a = 5 and d = 3 to get $a_n = 5 + (n-1)3$	**M1**	3.1b	5th Use arithmetic sequences and series in context.
	Simplifies to state $a_n = 3n + 2$	**A1**	1.1b	
		(2)		
(b)	Use the sum of an arithmetic series to state $\frac{k}{2}\left[10 + (k-1)3\right] = 948$	**M1**	3.1b	5th Use arithmetic sequences and series in context.
	States correct final answer $3k^2 + 7k - 1896 = 0$	**A1**	1.1b	
		(2)		
				(4 marks)

Notes

15	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Understands that integration is required to solve the problem. For example, writes $\int_{\frac{\pi}{2}}^{\pi}\left(x\sin^2 x\right)\mathrm{d}x$	**M1**	3.1a	6th Use definite integration to find areas between curves.
	Uses the trigonometric identity $\cos 2x \equiv 1 - 2\sin^2 x$ to rewrite $\int_{\frac{\pi}{2}}^{\pi} x\sin^2 x\,\mathrm{d}x$ as $\int_{\frac{\pi}{2}}^{\pi}\left(\frac{1}{2}x - \frac{1}{2}x\cos 2x\right)\mathrm{d}x$ o.e.	**M1**	2.2a	
	Shows $\int_{\frac{\pi}{2}}^{\pi}\frac{1}{2}x\,\mathrm{d}x = \left[\frac{1}{4}x^2\right]_{\frac{\pi}{2}}^{\pi}$	**A1**	1.1b	
	Demonstrates an understanding of the need to find $\int_{\frac{\pi}{2}}^{\pi}\frac{1}{2}x\cos 2x\,\mathrm{d}x$ using integration by parts. For example, $u = x, \frac{\mathrm{d}u}{\mathrm{d}x} = 1$ $\frac{\mathrm{d}v}{\mathrm{d}x} = \cos 2x, v = \frac{1}{2}\sin 2x$ o.e. is seen.	**M1**	2.2a	
	States fully correct integral $\int_{\frac{\pi}{2}}^{\pi}\left(\frac{1}{2}x - \frac{1}{2}x\cos 2x\right)\mathrm{d}x = \left[\frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x\right]_{\frac{\pi}{2}}^{\pi}$	**A1**	1.1b	
	Makes an attempt to substitute the limits $\left(\frac{\pi^2}{4} - \frac{1}{4}(0) - \frac{1}{8}(1)\right) - \left(\frac{\pi^2}{16} - \frac{1}{4}(0) - \frac{1}{8}(-1)\right)$	**M1**	2.2a	
	States fully correct answer: either $\frac{3\pi^2}{16} - \frac{1}{4}$ or $\frac{3\pi^2 - 4}{16}$ o.e.	**A1**	1.1b	
				(7 marks)

Notes

Integration shown without the limits is acceptable for earlier method and accuracy marks. Must correctly substitute limits at step 6