Pearson Edexcel Level 3

GCE Mathematics

Advanced Level

Paper 1 or 2: Pure Mathematics

Practice Paper A **Time: 2 hours**	**Paper Reference(s)** **9MA0/01 or 9MA0/02**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Answer ALL questions.

1. It is suggested that the sequence $a_k = 2^k + 1, k \ldots 1$ produces only prime numbers.

(a) Show that a_1, a_2 and a_4 produce prime numbers.

(2 marks)

(b) Prove by counter example that the sequence does not always produce a prime number.

(2 marks)

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2. Find the angle that the vector $\mathbf{a} = 4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ makes with the positive y-axis.

(3 marks)

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3. $$\mathrm{g}(x) \quad 3\sin \quad \frac{x}{6}^{3} \quad \frac{1}{10}x \quad 1$$, $-40 < x < 20$, x is in radians.

(a) Show that the equation $\mathrm{g}(x) = 0$ can be written as $x \quad 6 \sqrt[3]{\arcsin \quad \frac{1}{3} \quad \frac{1}{30}x}$

(3 marks)

(b) Using the formula $x_{n\ 1} \quad 6 \sqrt[3]{\arcsin \quad \frac{1}{3} \quad \frac{1}{30}x_n}$, $x_0 \quad 4$, find, to 3 decimal places, the values of x_1, x_2 and x_3.

(2 marks)

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4. The first 3 terms of a geometric sequence are $k + 2, 4k, 2k^2$, $k > 0$. Find the value of k.

(4 marks)

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5. $$\mathrm{f}(x) = \frac{x^4 + 2x^3 - 29x^2 - 47x + 77}{x^2 - 2x - 15}$$

Show that $\mathrm{f}(x)$ can be written as $Px^2 \quad Qx \quad R \quad \frac{V}{x \quad 3} \quad \frac{W}{x \quad 5}$ and find the values of P, Q, R, V and W.

(7 marks)

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6. Figure 1 shows a logo comprised of a rhombus surrounded by two arcs. Arc BAD has centre C and arc BCD has centre A. Some of the dimensions of the logo are shown in the diagram.

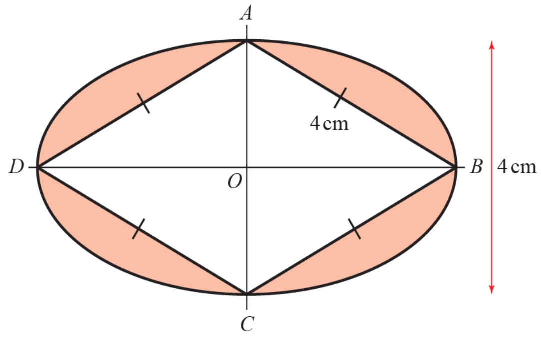

Figure 1

Prove that the shaded area of the logo is $\frac{2}{3}\left(16\pi - 24\sqrt{3}\right)$

(8 marks)

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7. C has parametric equations $x = \dfrac{1+4t}{1-t}$, $y = \dfrac{2+bt}{1-t}$, $-1 \leq t \leq 0$.

(a) Show that the cartesian equation of C is $y = \left(\dfrac{2+b}{5}\right)x + \left(\dfrac{8-b}{5}\right)$, over an appropriate domain.

(4 marks)

Given that C is a line segment and that the gradient of the line is -1,

(b) show that the length of the line segment is $a\sqrt{2}$, where a is a rational number to be found.

(4 marks)

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8. A toy soldier is connected to a parachute. The soldier is thrown into the air from ground level. The height, in metres, of the soldier above the ground can be modelled by the equation $H = \dfrac{4t^{\frac{2}{3}}}{t^2+1}$, $0 \leq t \leq 6$ s, where H is height of the soldier above the ground and t is the time since the soldier was thrown.

(a) Show that $\dfrac{\mathrm{d}H}{\mathrm{d}t} = \dfrac{8\left(1-2t^2\right)}{3\sqrt[3]{t}\left(t^2+1\right)^2}$.

(4 marks)

(b) Using the differentiated function, explain whether the soldier was increasing or decreasing in height after 2 seconds.

(2 marks)

(c) Find the exact time when the soldier reaches a maximum height.

(2 marks)

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9. (a) Show that $\tan^4 x \equiv \sec^2 x \tan^2 x + 1 - \sec^2 x$.

(4 marks)

(b) Hence find the exact value of $\int_0^{\frac{\pi}{4}} \tan^4 x \, dx$.

(5 marks)

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10. Use proof by contradiction to show that, given a rational number a and an irrational number b, $a - b$ is irrational.

(4 marks)

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11. $$\mathrm{f}(x) = |2x+3| - 4, \; x \in \mathbb{R}.$$

(a) Sketch the graph of $y = \mathrm{f}(x)$, labelling its vertex and any points of intersection with the coordinate axes.

(5 marks)

(b) Find the coordinates of the points of intersection of $y = |2x+3| - 4$ and $y = -\frac{1}{4}x + 2$.

(5 marks)

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12. (a) Prove that $(\sin 3 \quad \cos 3\ )^2 \quad 1 \quad \sin 6$

(3 marks)

(b) Use the result to solve, for $0 „ \theta „ \frac{\pi}{2}$, the equation $(\sin 3 \quad \cos 3\ ) \quad \sqrt{\frac{2 \quad \sqrt{2}}{2}}$.
Give your answer in terms of π. Check for extraneous solutions.

(4 marks)

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13. $$\mathrm{f}(x) \quad \frac{6}{2 \quad 3x} \quad \frac{4}{3 \quad 5x}, \; |x| \quad \frac{3}{5}.$$

(a) Show that the first three terms in the series expansion of $\mathrm{f}(x)$ can be written as $\frac{5}{3} - \frac{121}{18}x + \frac{329}{108}x^2$.

(7 marks)

(b) Find the exact value of $\mathrm{f}(0.01)$. Round your answer to 7 decimal places.

(2 marks)

(c) Find the percentage error made in using the series expansion in part (a) to estimate the value of $\mathrm{f}(0.01)$.

Give your answer to 2 significant figures.

(3 marks)

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14. Jacob is making some patterns out of squares. The first 3 patterns in the sequence are shown in Figure 2.

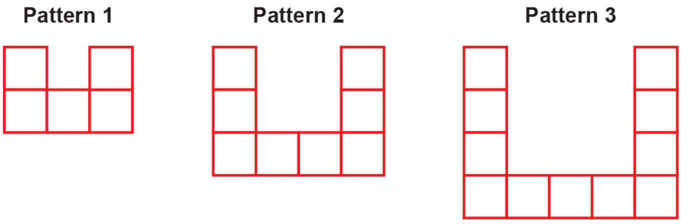

Figure 2

(a) Find an expression, in terms of n, for the number of squares required to make pattern n.

(2 marks)

Jacob uses a total of 948 squares in constructing the first k patterns.

(b) Show that $3k^2 + 7k - 1896 = 0$.

(2 marks)

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15. Figure 3 shows part of the curve with equation $y = x\sin^2 x$. The finite region bounded by the line with equation $x = \frac{\pi}{2}$, the curve and the x-axis is shown shaded in the diagram.

Find the area of the shaded region.

(7 marks)

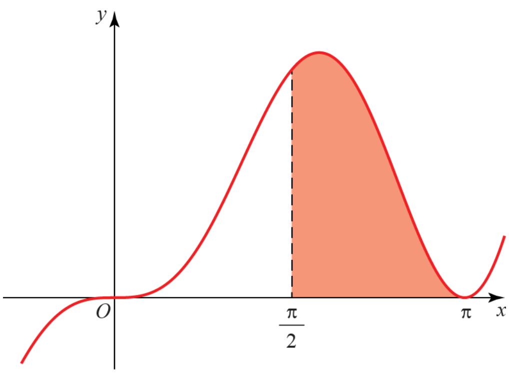

Figure 3

TOTAL FOR PAPER IS 100 MARKS

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