**1.**

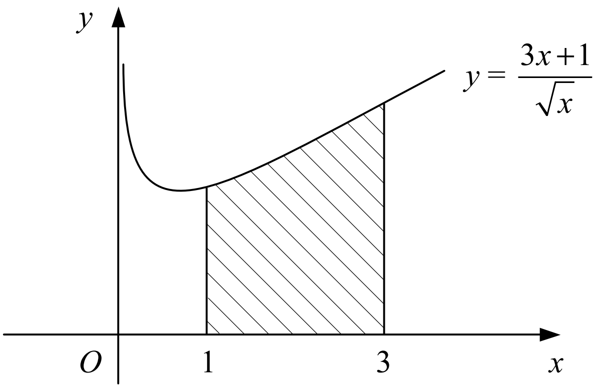


**Figure 1**

Figure 1 shows the curve with equation $y = \dfrac{3x+1}{\sqrt{x}}$, $x > 0$.

The shaded region is bounded by the curve, the $x$-axis and the lines $x = 1$ and $x = 3$.

Find the volume of the solid formed when the shaded region is rotated through $2\pi$ radians about the $x$-axis, giving your answer in the form $\pi(a + \ln b)$, where $a$ and $b$ are integers. **(6)**

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**2.** *(a)* Expand $(1 - 3x)^{-2}$, $|x| < \frac{1}{3}$, in ascending powers of $x$ up to and including the term in $x^3$, simplifying each coefficient. **(4)**

*(b)* Hence, or otherwise, show that for small $x$,

$$\left(\frac{2-x}{1-3x}\right)^2 \approx 4 + 20x + 85x^2 + 330x^3.$$ **(3)**

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**3.**
$$\mathrm{f}(x) = \frac{7+3x+2x^2}{(1-2x)(1+x)^2}, \quad |x| > \tfrac{1}{2}.$$

*(a)* Express f($x$) in partial fractions. **(4)**

*(b)* Show that

$$\int_1^2 \mathrm{f}(x)\ \mathrm{d}x = p - \ln q,$$

where $p$ is rational and $q$ is an integer. **(7)**

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**4.** Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 4 \\ -2 \end{pmatrix}$$

and $$\mathbf{r} = \begin{pmatrix} a \\ 6 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 14 \\ 2 \end{pmatrix},$$

where $a$ is a constant and $\lambda$ and $\mu$ are scalar parameters.

Given that the two lines intersect,

*(a)* find the position vector of their point of intersection, **(5)**

*(b)* find the value of $a$. **(2)**

Given also that $\theta$ is the acute angle between the lines,

*(c)* find the value of $\cos \theta$ in the form $k\sqrt{5}$ where $k$ is rational. **(4)**

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**5.** A curve has the equation

$$x^2 - 4xy + 2y^2 = 1.$$

*(a)* Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form in terms of $x$ and $y$. **(5)**

*(b)* Show that the tangent to the curve at the point $P$ (1, 2) has the equation

$$3x - 2y + 1 = 0.$$ **(3)**

The tangent to the curve at the point $Q$ is parallel to the tangent at $P$.

*(c)* Find the coordinates of $Q$. **(4)**

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***Turn over***

6. The rate of increase in the number of bacteria in a culture, $N$, at time $t$ hours is proportional to $N$.

(a) Write down a differential equation connecting $N$ and $t$. **(1)**

Given that initially there are $N_0$ bacteria present in a culture,

(b) Show that $N = N_0 e^{kt}$, where $k$ is a positive constant. **(6)**

Given also that the number of bacteria present doubles every six hours,

(c) find the value of $k$, **(3)**

(d) find how long it takes for the number of bacteria to increase by a factor of ten, giving your answer to the nearest minute. **(3)**

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7. A curve has parametric equations

$$x = \sec\theta + \tan\theta, \quad y = \operatorname{cosec}\theta + \cot\theta, \quad 0 < \theta < \frac{\pi}{2}.$$

(a) Show that $x + \frac{1}{x} = 2\sec\theta$. **(5)**

Given that $y + \frac{1}{y} = 2\operatorname{cosec}\theta$,

(b) find a cartesian equation for the curve. **(3)**

(c) Show that $\frac{dx}{d\theta} = \frac{1}{2}(x^2 + 1)$. **(3)**

(d) Find an expression for $\frac{dy}{dx}$ in terms of $x$ and $y$. **(4)**

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**END**

# C4 Paper K – Marking Guide

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|---|---|---|---|
| **1.** | $= \pi \int_1^3 \frac{(3x+1)^2}{x}\, dx$ | M1 | |
| | $= \pi \int_1^3 \frac{9x^2+6x+1}{x}\, dx = \int_1^3 (9x + 6 + \frac{1}{x})\, dx$ | A1 | |
| | $= \pi[\frac{9}{2}x^2 + 6x + \ln\lvert x \rvert]_1^3$ | M1 A1 | |
| | $= \pi\{(\frac{81}{2} + 18 + \ln 3) - (\frac{9}{2} + 6 + 0)\}$ | M1 | |
| | $= \pi(48 + \ln 3)$ | A1 | **(6)** |

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|---|---|---|---|
| **2.** | *(a)* $(1 - 3x)^{-2} = 1 + (-2)(-3x) + \frac{(-2)(-3)}{2}(-3x)^2 + \frac{(-2)(-3)(-4)}{3\times 2}(-3x)^3 + \ldots$ | M1 | |
| | $= 1 + 6x + 27x^2 + 108x^3 + \ldots$ | A3 | |
| | *(b)* $\left(\frac{2-x}{1-3x}\right)^2 = (2 - x)^2(1 - 3x)^{-2} = (4 - 4x + x^2)(1 + 6x + 27x^2 + 108x^3 + \ldots)$ | M1 | |
| | $= 4 + 24x + 108x^2 + 432x^3 - 4x - 24x^2 - 108x^3 + x^2 + 6x^3 + \ldots$ | A1 | |
| | $\therefore$ for small $x$, $\left(\frac{2-x}{1-3x}\right)^2 = 4 + 20x + 85x^2 + 330x^3$ | A1 | **(7)** |

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|---|---|---|---|
| **3.** | *(a)* $\frac{7+3x+2x^2}{(1-2x)(1+x)^2} \equiv \frac{A}{1-2x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$ | | |
| | $7 + 3x + 2x^2 \equiv A(1 + x)^2 + B(1 - 2x)(1 + x) + C(1 - 2x)$ | | |
| | $x = \frac{1}{2} \;\Rightarrow\; 9 = \frac{9}{4}A \;\Rightarrow\; A = 4$ | B1 | |
| | $x = -1 \;\Rightarrow\; 6 = 3C \;\Rightarrow\; C = 2$ | B1 | |
| | coeffs $x^2 \;\Rightarrow\; 2 = A - 2B \;\Rightarrow\; B = 1$ | M1 | |
| | $\therefore\; f(x) = \frac{4}{1-2x} + \frac{1}{1+x} + \frac{2}{(1+x)^2}$ | A1 | |
| | *(b)* $= \int_1^2 (\frac{4}{1-2x} + \frac{1}{1+x} + \frac{2}{(1+x)^2})\, dx$ | | |
| | $= [-2 \ln\lvert 1 - 2x\rvert + \ln\lvert 1 + x\rvert - 2(1 + x)^{-1}]_1^2$ | M1 A3 | |
| | $= (-2 \ln 3 + \ln 3 - \frac{2}{3}) - (0 + \ln 2 - 1)$ | M1 | |
| | $= -\ln 3 - \ln 2 + \frac{1}{3} = \frac{1}{3} - \ln 6 \qquad [\, p = \frac{1}{3}, q = 6\, ]$ | M1 A1 | **(11)** |

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| **4.** | *(a)* $4\lambda = 6 + 14\mu \quad (1)$ | | |
| | $-3 - 2\lambda = 3 + 2\mu \quad (2)$ | B1 | |
| | $(1) + 2 \times (2)$: $-6 = 12 + 18\mu,\; \mu = -1,\; \lambda = -2$ | M1 A1 | |
| | $\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ -3 \end{pmatrix} - 2\begin{pmatrix} 5 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \\ 1 \end{pmatrix}$ | M1 A1 | |
| | *(b)* $a - (-5) = -3, \quad a = -8$ | M1 A1 | |
| | *(c)* $\cos\theta = \left\lvert \frac{5\times(-5)+4\times 14+(-2)\times 2}{\sqrt{25+16+4}\times\sqrt{25+196+4}} \right\rvert$ | M1 A1 | |
| | $= \frac{27}{\sqrt{45\times 15}} = \frac{9}{3\sqrt{5}\times 5} = \frac{3}{5\sqrt{5}} = \frac{3}{25}\sqrt{5}$ | M1 A1 | **(11)** |

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| **5.** | *(a)* | $2x - 4y - 4x\dfrac{dy}{dx} + 4y\dfrac{dy}{dx} = 0$ | M1 A2 | |
| | | $\dfrac{dy}{dx} = \dfrac{2x-4y}{4x-4y} = \dfrac{x-2y}{2x-2y}$ | M1 A1 | |
| | *(b)* | grad $= \frac{3}{2}$ | M1 | |
| | | $\therefore\ y - 2 = \frac{3}{2}(x-1)$ | M1 | |
| | | $2y - 4 = 3x - 3$ | | |
| | | $3x - 2y + 1 = 0$ | A1 | |
| | *(c)* | $\dfrac{x-2y}{2x-2y} = \frac{3}{2}$ | M1 | |
| | | $2(x-2y) = 3(2x-2y),\quad y = 2x$ | A1 | |
| | | sub. $\Rightarrow\ x^2 - 8x^2 + 8x^2 = 1$ | M1 | |
| | | $x^2 = 1,\ \ x = 1$ (at $P$) or $-1$ | | |
| | | $\therefore\ Q(-1, -2)$ | A1 | **(12)** |
| **6.** | *(a)* | $\dfrac{dN}{dt} = kN$ | B1 | |
| | *(b)* | $\int \frac{1}{N}\ dN = \int k\ dt$ | M1 | |
| | | $\ln\lvert N\rvert = kt + c$ | M1 A1 | |
| | | $t = 0,\ N = N_0 \Rightarrow \ln\lvert N_0\rvert = c$ | M1 | |
| | | $\ln\lvert N\rvert = kt + \ln\lvert N_0\rvert,\quad \ln\left\lvert \dfrac{N}{N_0}\right\rvert = kt$ | M1 | |
| | | $\dfrac{N}{N_0} = e^{kt},\qquad N = N_0e^{kt}$ | A1 | |
| | *(c)* | $2N_0 = N_0e^{6k}$ | M1 | |
| | | $k = \frac{1}{6}\ln 2 = 0.116$ (3sf) | M1 A1 | |
| | *(d)* | $10N_0 = N_0e^{0.1155t}$ | M1 | |
| | | $t = \frac{1}{0.1155}\ln 10 = 19.932$ hours $= 19$ hours $56$ mins | M1 A1 | **(13)** |
| **7.** | *(a)* | $x + \dfrac{1}{x} = \sec\theta + \tan\theta + \dfrac{1}{\sec\theta+\tan\theta} = \dfrac{(\sec\theta+\tan\theta)^2+1}{\sec\theta+\tan\theta}$ | M1 | |
| | | $= \dfrac{\sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta + 1}{\sec\theta+\tan\theta} = \dfrac{2\sec^2\theta + 2\sec\theta\tan\theta}{\sec\theta+\tan\theta}$ | M1 A1 | |
| | | $= \dfrac{2\sec\theta(\sec\theta+\tan\theta)}{\sec\theta+\tan\theta} = 2\sec\theta$ | M1 A1 | |
| | *(b)* | $\dfrac{x^2+1}{x} = \dfrac{2}{\cos\theta} \Rightarrow \cos\theta = \dfrac{2x}{x^2+1}$ | M1 | |
| | | $\dfrac{y^2+1}{y} = \dfrac{2}{\sin\theta} \Rightarrow \sin\theta = \dfrac{2y}{y^2+1}\quad \therefore\ \dfrac{4x^2}{(x^2+1)^2} + \dfrac{4y^2}{(y^2+1)^2} = 1$ | M1 A1 | |
| | *(c)* | $\dfrac{dx}{d\theta} = \sec\theta\tan\theta + \sec^2\theta$ | M1 | |
| | | $= \sec\theta(\tan\theta + \sec\theta) = \dfrac{x^2+1}{2x} \times x = \frac{1}{2}(x^2+1)$ | M1 A1 | |
| | *(d)* | $\dfrac{dy}{d\theta} = -\text{cosec}\,\theta\cot\theta - \text{cosec}^2\,\theta$ | M1 | |
| | | $= -\text{cosec}\,\theta(\cot\theta + \text{cosec}\,\theta) = -\dfrac{y^2+1}{2y} \times y = -\frac{1}{2}(y^2+1)$ | A1 | |
| | | $\therefore\ \dfrac{dy}{dx} = -\dfrac{y^2+1}{x^2+1}$ | M1 A1 | **(15)** |
| | | | Total | **(75)** |