1. The region bounded by the curve $y = x^2 - 2x$ and the x-axis is rotated through 2π radians about the x-axis.

Find the volume of the solid formed, giving your answer in terms of π. **(6)**

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2. Use the substitution $u = 1 - x^{\frac{1}{2}}$ to find

$$\int \frac{1}{1 - x^{\frac{1}{2}}} \, \mathrm{d}x.$$ **(6)**

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3. A curve has the equation

$$2 \sin 2x - \tan y = 0.$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form in terms of x and y. **(5)**

(b) Show that the tangent to the curve at the point $(\frac{\pi}{6}, \frac{\pi}{3})$ has the equation

$$y = \tfrac{1}{2}x + \tfrac{\pi}{4}.$$ **(3)**

4.

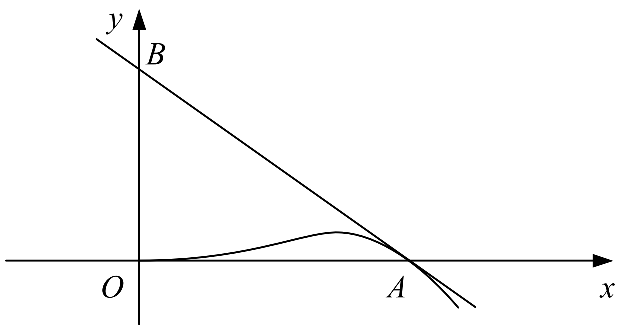

Figure 1

Figure 1 shows the curve with parametric equations

$$x = a\sqrt{t}, \quad y = at(1 - t), \quad t \geq 0,$$

where a is a positive constant.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. **(3)**

The curve meets the x-axis at the origin, O, and at the point A. The tangent to the curve at A meets the y-axis at the point B as shown.

(b) Show that the area of triangle OAB is a^2. **(6)**

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5. The gradient at any point (x, y) on a curve is proportional to $\sqrt{y}$.

Given that the curve passes through the point with coordinates (0, 4),

(a) show that the equation of the curve can be written in the form

$$2\sqrt{y} = kx + 4,$$

where k is a positive constant. **(5)**

Given also that the curve passes through the point with coordinates (2, 9),

(b) find the equation of the curve in the form $y = \mathrm{f}(x)$. **(4)**

6.

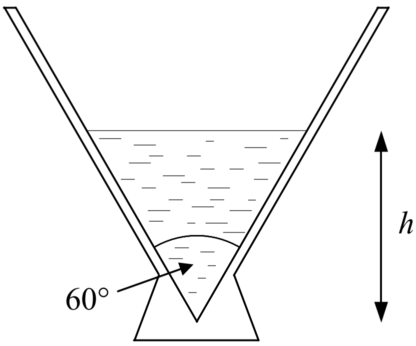

Figure 2

Figure 2 shows a vertical cross-section of a vase.

The inside of the vase is in the shape of a right-circular cone with the angle between the sides in the cross-section being 60°. When the depth of water in the vase is h cm, the volume of water in the vase is V cm^3.

(a) Show that $V = \frac{1}{9}\pi h^3$. **(3)**

The vase is initially empty and water is poured in at a constant rate of 120 cm^3 s^{-1}.

(b) Find, to 2 decimal places, the rate at which h is increasing

(i) when $h = 6$,

(ii) after water has been poured in for 8 seconds. **(7)**

Turn over

7. Relative to a fixed origin, the points A and B have position vectors $\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 6 \\ 1 \end{pmatrix}$ respectively.

(a) Find a vector equation for the line l_1 which passes through A and B. **(2)**

The line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ -7 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}.$$

(b) Show that lines l_1 and l_2 do not intersect. **(5)**

(c) Find the position vector of the point C on l_2 such that $\angle ABC = 90°$. **(6)**

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8. $$\mathrm{f}(x) = \frac{x(3x-7)}{(1-x)(1-3x)}, \quad |x| < \tfrac{1}{3}.$$

(a) Find the values of the constants A, B and C such that

$$\mathrm{f}(x) = A + \frac{B}{1-x} + \frac{C}{1-3x}.$$ **(4)**

(b) Evaluate

$$\int_0^{\frac{1}{4}} \mathrm{f}(x)\ \mathrm{d}x,$$

giving your answer in the form $p + \ln q$, where p and q are rational. **(5)**

(c) Find the series expansion of $\mathrm{f}(x)$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(5)**

END

C4 Paper J – Marking Guide

1. $x(x-2)=0,\ x=0,2\ \therefore$ crosses x-axis at $(0, 0)$ and $(2, 0)$

volume $= \pi\int_0^2 (x^2-2x)^2\ \mathrm{d}x$ — M1

$= \pi\int_0^2 (x^4-4x^3+4x^2)\ \mathrm{d}x$ — A1

$= \pi[\tfrac{1}{5}x^5 - x^4 + \tfrac{4}{3}x^3]_0^2$ — M1 A1

$= \pi\{(\tfrac{32}{5} - 16 + \tfrac{32}{3}) - (0)\} = \tfrac{16}{15}\pi$ — M1 A1 **(6)**

---

2. $u = 1 - x^{\frac{1}{2}} \Rightarrow x = (1-u)^2,\ \dfrac{\mathrm{d}x}{\mathrm{d}u} = -2(1-u) = 2u-2$ — M1 A1

$I = \int \frac{1}{u}\times(2u-2)\ \mathrm{d}u = \int (2 - \frac{2}{u})\ \mathrm{d}u$ — A1

$= 2u - 2\ln|u| + c$ — M1 A1

$= 2(1-x^{\frac{1}{2}}) - 2\ln|1-x^{\frac{1}{2}}| + c$ — A1 **(6)**

---

3. *(a)* $4\cos 2x - \sec^2 y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ — M1 A2

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4\cos 2x\cos^2 y$ — M1 A1

(b) grad $= 4\times\tfrac{1}{2}\times\tfrac{1}{4} = \tfrac{1}{2}$ — B1

$\therefore\ y - \tfrac{\pi}{3} = \tfrac{1}{2}(x - \tfrac{\pi}{6})$ — M1

$y - \tfrac{\pi}{3} = \tfrac{1}{2}x - \tfrac{\pi}{12}$

$y = \tfrac{1}{2}x + \tfrac{\pi}{4}$ — A1 **(8)**

---

4. *(a)* $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \tfrac{1}{2}at^{-\frac{1}{2}},\quad \dfrac{\mathrm{d}y}{\mathrm{d}t} = a(1-2t)$ — M1

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{a(1-2t)}{\frac{1}{2}at^{-\frac{1}{2}}} = 2\sqrt{t}\,(1-2t)$ — M1 A1

(b) $y = 0 \Rightarrow t = 0$ (at O) or 1 (at A) — B1

$t = 1,\ x = a,\ y = 0,$ grad $= -2$ — M1

$\therefore\ y - 0 = -2(x-a)$ — A1

at B, $x = 0\ \therefore y = 2a$ — M1

area $= \tfrac{1}{2}\times a\times 2a = a^2$ — M1 A1 **(9)**

---

5. *(a)* $\dfrac{\mathrm{d}y}{\mathrm{d}x} = k\sqrt{y}$

$\int y^{-\frac{1}{2}}\ \mathrm{d}y = \int k\ \mathrm{d}x$ — M1

$2y^{\frac{1}{2}} = kx + c$ — M1 A1

$(0, 4) \Rightarrow 4 = c$ — M1

$\therefore\ 2\sqrt{y} = kx + 4$ — A1

(b) $(2, 9) \Rightarrow 6 = 2k + 4,\quad k = 1$ — M1 A1

$\therefore\ 2\sqrt{y} = x + 4,\quad \sqrt{y} = \tfrac{1}{2}(x+4)$ — M1

$y = \tfrac{1}{4}(x+4)^2$ — A1 **(9)**

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6. (a)	let radius = r, $\therefore \tan 30° = \frac{1}{\sqrt{3}} = \frac{r}{h}$, $r = \frac{h}{\sqrt{3}}$	M1	
	$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h \times \frac{h^2}{3} = \frac{1}{9}\pi h^3$	M1 A1	
(b) (i)	$\frac{dV}{dt} = 120$, $\frac{dV}{dh} = \frac{1}{3}\pi h^2$	B1	
	$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$, $120 = \frac{1}{3}\pi h^2 \frac{dh}{dt}$, $\frac{dh}{dt} = \frac{360}{\pi h^2}$	M1 A1	
	when $h = 6$, $\frac{dh}{dt} = 3.18 \text{ cm s}^{-1}$ (2dp)	M1 A1	
(ii)	$V = 8 \times 120 = 960 = \frac{1}{9}\pi h^3$ $\therefore h = \sqrt[3]{\frac{9\times 960}{\pi}} = 14.011$	M1	
	$\therefore \frac{dh}{dt} = 0.58 \text{ cm s}^{-1}$ (2dp)	A1	**(10)**

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7. (a)	$\overrightarrow{AB} = \begin{pmatrix}-3\\6\\1\end{pmatrix} - \begin{pmatrix}-4\\1\\3\end{pmatrix} = \begin{pmatrix}1\\5\\-2\end{pmatrix}$ $\therefore \mathbf{r} = \begin{pmatrix}-4\\1\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\5\\-2\end{pmatrix}$	M1 A1	
(b)	$-4 + \lambda = 3 + 2\mu$ (1)		
	$1 + 5\lambda = -7 - 3\mu$ (2)		
	$3 - 2\lambda = 9 + \mu$ (3)	B1	
	$2 \times (1) + (3)$: $-5 = 15 + 5\mu$, $\mu = -4$, $\lambda = -1$	M1 A1	
	sub. (2): $1 - 5 = -7 + 12$, not true $\therefore$ do not intersect	M1 A1	
(c)	$\overrightarrow{OC} = \begin{pmatrix}3+2\mu\\-7-3\mu\\9+\mu\end{pmatrix}$, $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix}6+2\mu\\-13-3\mu\\8+\mu\end{pmatrix}$	M1 A1	
	$\therefore \begin{pmatrix}1\\5\\-2\end{pmatrix} \cdot \begin{pmatrix}6+2\mu\\-13-3\mu\\8+\mu\end{pmatrix} = 0$, $6 + 2\mu - 65 - 15\mu - 16 - 2\mu = 0$	M1 A1	
	$\mu = -5$ $\therefore \overrightarrow{OC} = \begin{pmatrix}-7\\8\\4\end{pmatrix}$	M1 A1	**(13)**

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8. (a)	$x(3x - 7) \equiv A(1 - x)(1 - 3x) + B(1 - 3x) + C(1 - x)$	M1	
	$x = 1 \Rightarrow -4 = -2B \Rightarrow B = 2$	A1	
	$x = \frac{1}{3} \Rightarrow -2 = \frac{2}{3}C \Rightarrow C = -3$	A1	
	coeffs $x^2 \Rightarrow 3 = 3A \Rightarrow A = 1$	A1	
(b)	$= \int_0^{\frac{1}{4}} (1 + \frac{2}{1-x} - \frac{3}{1-3x})\, dx = [x - 2\ln\lvert 1 - x\rvert + \ln\lvert 1 - 3x\rvert]_0^{\frac{1}{4}}$	M1 A1	
	$= (\frac{1}{4} - 2\ln\frac{3}{4} + \ln\frac{1}{4}) - (0)$	M1	
	$= \frac{1}{4} + \ln\frac{16}{9} + \ln\frac{1}{4} = \frac{1}{4} + \ln\frac{4}{9}$	M1 A1	
(c)	$f(x) = 1 + 2(1 - x)^{-1} - 3(1 - 3x)^{-1}$		
	$(1 - x)^{-1} = 1 + x + x^2 + x^3 + \ldots$	B1	
	$(1 - 3x)^{-1} = 1 + 3x + (3x)^2 + (3x)^3 + \ldots = 1 + 3x + 9x^2 + 27x^3 + \ldots$	M1 A1	
	$\therefore f(x) = 1 + 2(1 + x + x^2 + x^3 + \ldots) - 3(1 + 3x + 9x^2 + 27x^3 + \ldots)$	M1	
	$= -7x - 25x^2 - 79x^3 + \ldots$	A1	**(14)**

Total **(75)**