1. A curve has the equation

$$x^3 + 2xy - y^2 + 24 = 0.$$

Show that the normal to the curve at the point $(2, -4)$ has the equation $y = 3x - 10$. **(8)**

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2. *(a)* Expand $(4 - x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2, simplifying each coefficient. **(4)**

(b) State the set of values of x for which your expansion is valid. **(1)**

(c) Use your expansion with $x = 0.01$ to find the value of $\sqrt{399}$, giving your answer to 9 significant figures. **(4)**

3.

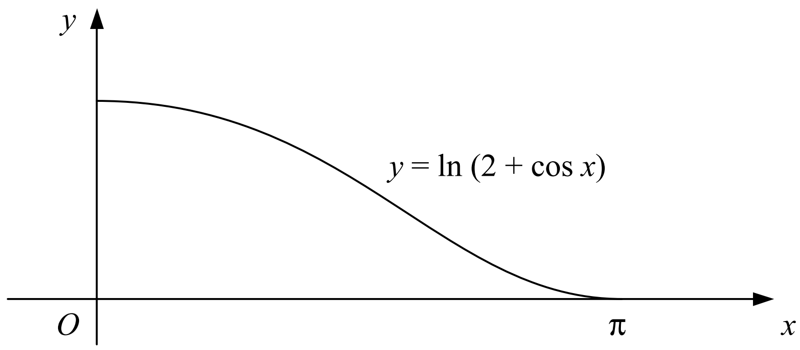

Figure 1

Figure 1 shows the curve with equation $y = \ln(2 + \cos x)$, $0 \le x \le \pi$.

(a) Copy and complete the table below for points on the curve, giving the y values to 4 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	1.0986				0

(2)

(b) Giving your answers to 3 decimal places, find estimates for the area of the region bounded by the curve and the coordinate axes using the trapezium rule with

(i) 1 strip,

(ii) 2 strips,

(iii) 4 strips. . **(6)**

(c) Making your reasoning clear, suggest a value to 2 decimal places for the actual area of the region bounded by the curve and the coordinate axes. **(2)**

4.

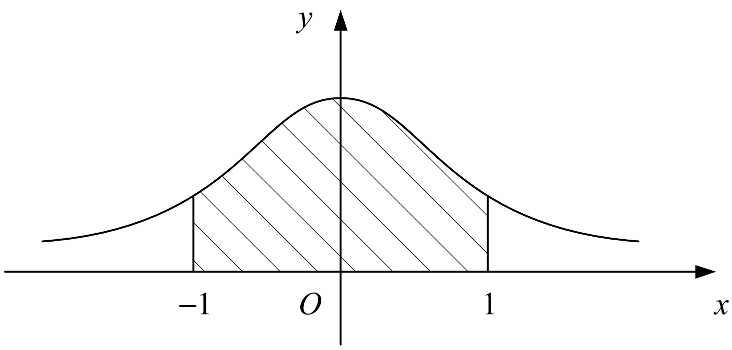

Figure 2

Figure 2 shows the curve with parametric equations

$$x = \tan\theta, \quad y = \cos^2\theta, \quad -\tfrac{\pi}{2} < \theta < \tfrac{\pi}{2}.$$

The shaded region bounded by the curve, the x-axis and the lines $x = -1$ and $x = 1$ is rotated through 2π radians about the x-axis.

(a) Show that the volume of the solid formed is $\frac{1}{4}\pi(\pi + 2)$. **(8)**

(b) Find a cartesian equation for the curve. **(3)**

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5. Relative to a fixed origin, the points A, B and C have position vectors $(2\mathbf{i} - \mathbf{j} + 6\mathbf{k})$, $(5\mathbf{i} - 4\mathbf{j})$ and $(7\mathbf{i} - 6\mathbf{j} - 4\mathbf{k})$ respectively.

(a) Show that A, B and C all lie on a single straight line. **(3)**

(b) Write down the ratio $AB : BC$ **(1)**

The point D has position vector $(3\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

(c) Show that AD is perpendicular to BD. **(4)**

(d) Find the exact area of triangle ABD. **(3)**

Turn over

6. *(a)* Use the substitution $x = 2\sin u$ to evaluate

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\ \mathrm{d}x.$$ **(5)**

(b) Use integration by parts to evaluate

$$\int_0^{\frac{\pi}{2}} x\cos x\ \mathrm{d}x.$$ **(6)**

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7. When a plague of locusts attacks a wheat crop, the proportion of the crop destroyed after t hours is denoted by x. In a model, it is assumed that the rate at which the crop is destroyed is proportional to $x(1-x)$.

A plague of locusts is discovered in a wheat crop when one quarter of the crop has been destroyed.

Given that the rate of destruction at this instant is such that if it remained constant, the crop would be completely destroyed in a further six hours,

(a) show that $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \frac{2}{3}x(1-x)$, **(4)**

(b) find the percentage of the crop destroyed three hours after the plague of locusts is first discovered. **(11)**

END

C4 Paper I – Marking Guide

1.		$3x^2 + 2y + 2x\dfrac{dy}{dx} - 2y\dfrac{dy}{dx} = 0$	M1 A2	
		$(2, -4) \Rightarrow 12 - 8 + 4\dfrac{dy}{dx} + 8\dfrac{dy}{dx} = 0, \quad \dfrac{dy}{dx} = -\frac{1}{3}$	M1 A1	
		grad of normal = 3	M1	
		$\therefore\ y + 4 = 3(x - 2)$	M1	
		$y = 3x - 10$	A1	**(8)**
2.	*(a)*	$= 4^{\frac{1}{2}}(1 - \frac{1}{4}x)^{\frac{1}{2}} = 2(1 - \frac{1}{4}x)^{\frac{1}{2}}$	B1	
		$= 2[1 + (\frac{1}{2})(-\frac{1}{4}x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(-\frac{1}{4}x)^2 + \ldots] = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + \ldots$	M1 A2	
	(b)	$\lvert x \rvert < 4$	B1	
	(c)	$x = 0.01 \Rightarrow (4 - x)^{\frac{1}{2}} = \sqrt{3.99} = \sqrt{\frac{399}{100}} = \frac{1}{10}\sqrt{399}$	M1	
		$x = 0.01 \Rightarrow (4 - x)^{\frac{1}{2}} \approx 2 - \frac{1}{400} - \frac{1}{640000} = 1.997498438$	M1	
		$\therefore\ \sqrt{399} \approx 10 \times 1.997498438 = 19.9749844$ (9sf)	M1 A1	**(9)**
3.	*(a)*	0.9959, 0.6931, 0.2569 (4dp)	B2	
	(b)	*(i)* $= \frac{1}{2} \times \pi \times (1.0986 + 0) = 1.726$ (3dp)	B1 M1 A1	
		(ii) $= \frac{1}{2} \times \frac{\pi}{2} \times [1.0986 + 0 + 2(0.6931)] = 1.952$ (3dp)	M1 A1	
		(iii) $= \frac{1}{2} \times \frac{\pi}{4} \times [1.0986 + 0 + 2(0.9959 + 0.6931 + 0.2569)]$		
		$= 1.960$ (3dp)	A1	
	(c)	1.96; large change from 1 to 2 strips but from 2 to 4 strips the change is less than 0.01 so the error in 4 strip value is likely to be less than 0.005	B2	**(10)**
4.	*(a)*	$x = -1 \Rightarrow \theta = -\frac{\pi}{4},\ x = 1 \Rightarrow \theta = \frac{\pi}{4}$	B1	
		$\dfrac{dx}{d\theta} = \sec^2\theta$	M1	
		volume $= \pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\cos^2\theta)^2 \times \sec^2\theta\ d\theta = \pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2\theta\ d\theta$	A1	
		$= \pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\frac{1}{2} + \frac{1}{2}\cos 2\theta)\ d\theta$	M1	
		$= \pi[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$	M1 A1	
		$= \pi[(\frac{\pi}{8} + \frac{1}{4}) - (-\frac{\pi}{8} - \frac{1}{4})]$	M1	
		$= \pi(\frac{\pi}{4} + \frac{1}{2}) = \frac{1}{4}\pi(\pi + 2)$	A1	
	(b)	$y = \cos^2\theta = \dfrac{1}{\sec^2\theta} = \dfrac{1}{1 + \tan^2\theta} \quad \therefore\ y = \dfrac{1}{1 + x^2}$	M2 A1	**(11)**

5.	*(a)*	$\overrightarrow{AB} = (5\mathbf{i} - 4\mathbf{j}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (3\mathbf{i} - 3\mathbf{j} - 6\mathbf{k})$	B1	
		$\overrightarrow{AC} = (7\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (5\mathbf{i} - 5\mathbf{j} - 10\mathbf{k}) = \frac{5}{3}\overrightarrow{AB}$	M1	
		$\therefore$ $\overrightarrow{AC}$ is parallel to $\overrightarrow{AB}$, also common point $\therefore$ single straight line	A1	
	(b)	$3 : 2$	B1	
	(c)	$\overrightarrow{AD} = (3\mathbf{i} + \mathbf{j} + 4\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	B1	
		$\overrightarrow{BD} = (3\mathbf{i} + \mathbf{j} + 4\mathbf{k}) - (5\mathbf{i} - 4\mathbf{j}) = (-2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k})$	B1	
		$\overrightarrow{AD} \cdot \overrightarrow{BD} = -2 + 10 - 8 = 0$ $\therefore$ perpendicular	M1 A1	
	(d)	$= \frac{1}{2} \times \sqrt{1+4+4} \times \sqrt{4+25+16} = \frac{1}{2} \times 3 \times 3\sqrt{5} = \frac{9}{2}\sqrt{5}$	M2 A1	**(11)**
6.	*(a)*	$x = 2\sin u \Rightarrow \frac{dx}{du} = 2\cos u$	M1	
		$x = 0 \Rightarrow u = 0,\ x = \sqrt{3} \Rightarrow u = \frac{\pi}{3}$	B1	
		$I = \int_0^{\frac{\pi}{3}} \frac{1}{2\cos u} \times 2\cos u\ du = \int_0^{\frac{\pi}{3}} 1\ du$	A1	
		$= [u]_0^{\frac{\pi}{3}} = \frac{\pi}{3} - 0 = \frac{\pi}{3}$	M1 A1	
	(b)	$u = x,\ u' = 1,\ v' = \cos x,\ v = \sin x$	M1	
		$I = [x\sin x]_0^{\frac{\pi}{2}} - \int \sin x\ dx$	A2	
		$= [x\sin x + \cos x]_0^{\frac{\pi}{2}}$	M1	
		$= (\frac{\pi}{2} + 0) - (0 + 1) = \frac{\pi}{2} - 1$	M1 A1	**(11)**
7.	*(a)*	when $x = \frac{1}{4}$, $\frac{dx}{dt} = \frac{3}{4} \div 6 = \frac{1}{8}$	M1 A1	
		$\frac{dx}{dt} = kx(1 - x)$ $\therefore$ $\frac{1}{8} = k \times \frac{1}{4} \times \frac{3}{4}$, $k = \frac{2}{3}$ $\therefore$ $\frac{dx}{dt} = \frac{2}{3}x(1 - x)$	M1 A1	
	(b)	$\int \frac{1}{x(1-x)}\ dx = \int \frac{2}{3}\ dt$	M1	
		$\frac{1}{x(1-x)} \equiv \frac{A}{x} + \frac{B}{1-x}$, $\quad 1 \equiv A(1 - x) + Bx$	M1	
		$x = 0 \Rightarrow A = 1$	A1	
		$x = 1 \Rightarrow B = 1$	A1	
		$\therefore \int (\frac{1}{x} + \frac{1}{1-x})\ dx = \int \frac{2}{3}\ dt$		
		$\ln\lvert x\rvert - \ln\lvert 1 - x\rvert = \frac{2}{3}t + c$	M1 A1	
		$t = 0,\ x = \frac{1}{4} \Rightarrow \ln\frac{1}{4} - \ln\frac{3}{4} = c,\quad c = \ln\frac{1}{3}$	M1 A1	
		$t = 3 \Rightarrow \ln\lvert x\rvert - \ln\lvert 1 - x\rvert = 2 + \ln\frac{1}{3}$		
		$\ln\left\lvert\frac{3x}{1-x}\right\rvert = 2,\quad \frac{3x}{1-x} = e^2$	M1	
		$3x = e^2(1 - x),\quad x(e^2 + 3) = e^2$	M1	
		$x = \frac{e^2}{e^2+3}$ $\therefore$ % destroyed $= \frac{e^2}{e^2+3} \times 100\% = 71.1\%$ (3sf)	A1	**(15)**
			Total	**(75)**