1. *(a)* Expand $(1 + 4x)^{\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(4)**

(b) State the set of values of x for which your expansion is valid. **(1)**

---

2. Use the substitution $u = 1 + \sin x$ to find the value of

$$\int_0^{\frac{\pi}{2}} \cos x\,(1 + \sin x)^3 \; dx.$$ **(6)**

---

3. *(a)* Express $\dfrac{x+11}{(x+4)(x-3)}$ as a sum of partial fractions. **(3)**

(b) Evaluate

$$\int_0^2 \frac{x+11}{(x+4)(x-3)} \; dx,$$

giving your answer in the form $\ln k$, where k is an exact simplified fraction. **(5)**

4.

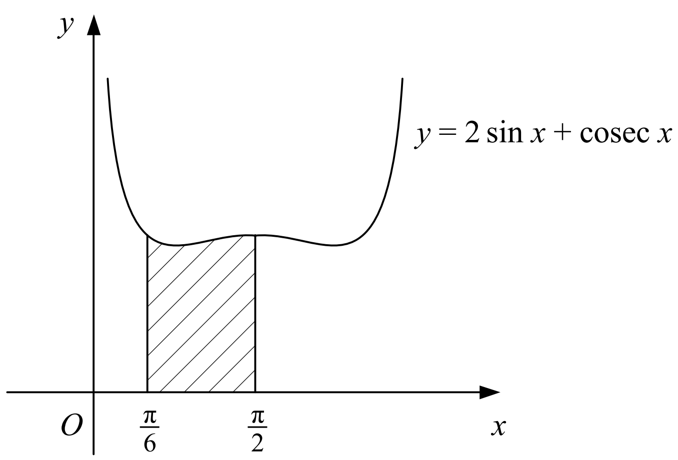

Figure 1

Figure 1 shows the curve with equation $y = 2\sin x + \operatorname{cosec} x$, $0 < x < \pi$.

The shaded region bounded by the curve, the x-axis and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$ is rotated through 360° about the x-axis.

Show that the volume of the solid formed is $\frac{1}{2}\pi(4\pi + 3\sqrt{3})$. **(8)**

---

5. A curve has the equation

$$x^2 - 3xy - y^2 = 12.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y. **(5)**

(b) Find an equation for the tangent to the curve at the point (2, –2). **(3)**

---

6. Relative to a fixed origin, O, the points A and B have position vectors $\begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$ respectively.

Find, in exact, simplified form,

(a) the cosine of $\angle AOB$, **(4)**

(b) the area of triangle OAB, **(4)**

(c) the shortest distance from A to the line OB. **(2)**

---

7. A curve has parametric equations

$$x = t(t - 1), \quad y = \frac{4t}{1-t}, \quad t \neq 1.$$

(a) Find $\frac{dy}{dx}$ in terms of t. **(4)**

The point P on the curve has parameter $t = -1$.

(b) Show that the tangent to the curve at P has the equation

$$x + 3y + 4 = 0.$$ **(3)**

The tangent to the curve at P meets the curve again at the point Q.

(c) Find the coordinates of Q. **(7)**

Turn over

8. An entomologist is studying the population of insects in a colony.

Initially there are 300 insects in the colony and in a model, the entomologist assumes that the population, P, at time t weeks satisfies the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP,$$

where k is a constant.

(a) Find an expression for P in terms of k and t. **(5)**

Given that after one week there are 360 insects in the colony,

(b) find the value of k to 3 significant figures. **(2)**

Given also that after two and three weeks there are 440 and 600 insects respectively,

(c) comment on suitability of the model. **(2)**

An alternative model assumes that

$$\frac{\mathrm{d}P}{\mathrm{d}t} = P(0.4 - 0.25 \cos 0.5t).$$

(d) Using the initial data, $P = 300$ when $t = 0$, solve this differential equation. **(4)**

(e) Compare the suitability of the two models. **(3)**

END

C4 Paper H – Marking Guide

1.	*(a)*	$= 1 + (\tfrac{3}{2})(4x) + \frac{(\frac{3}{2})(\frac{1}{2})}{2}(4x)^2 + \frac{(\frac{3}{2})(\frac{1}{2})(-\frac{1}{2})}{3\times 2}(4x)^3 + \ldots$	M1	
		$= 1 + 6x + 6x^2 - 4x^3 + \ldots$	A3	
	(b)	$\lvert x \rvert < \tfrac{1}{4}$	B1	**(5)**

2.	$u = 1 + \sin x \;\Rightarrow\; \dfrac{du}{dx} = \cos x$	M1	
	$x = 0 \;\Rightarrow\; u = 1,\; x = \tfrac{\pi}{2} \;\Rightarrow\; u = 2$	B1	
	$I = \int_1^2 u^3 \, du$	A1	
	$= [\tfrac{1}{4}u^4]_1^2$	M1	
	$= 4 - \tfrac{1}{4} = \tfrac{15}{4}$	M1 A1	**(6)**

3.	*(a)*	$\dfrac{x+11}{(x+4)(x-3)} \equiv \dfrac{A}{x+4} + \dfrac{B}{x-3}$		
		$x + 11 \equiv A(x - 3) + B(x + 4)$	M1	
		$x = -4 \;\Rightarrow\; 7 = -7A \;\Rightarrow\; A = -1$	A1	
		$x = 3 \;\Rightarrow\; 14 = 7B \;\Rightarrow\; B = 2$	A1	
		$\dfrac{x+11}{(x+4)(x-3)} \equiv \dfrac{2}{x-3} - \dfrac{1}{x+4}$		
	(b)	$= \int_0^2 \left(\dfrac{2}{x-3} - \dfrac{1}{x+4}\right) dx$		
		$= [2\ln\lvert x-3\rvert - \ln\lvert x+4\rvert]_0^2$	M1 A1	
		$= (0 - \ln 6) - (2\ln 3 - \ln 4)$	M1	
		$= \ln \tfrac{2}{27}$	M1 A1	**(8)**

4.	$= \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2\sin x + \operatorname{cosec} x)^2 \, dx$	M1	
	$= \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (4\sin^2 x + 4 + \operatorname{cosec}^2 x) \, dx$	A1	
	$= \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 - 2\cos 2x + 4 + \operatorname{cosec}^2 x) \, dx$	M1	
	$= \pi[6x - \sin 2x - \cot x]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$	M1 A2	
	$= \pi\{(3\pi + 0 + 0) - (\pi - \tfrac{\sqrt{3}}{2} - \sqrt{3})\}$	M1	
	$= \pi(2\pi + \tfrac{3}{2}\sqrt{3}) = \tfrac{1}{2}\pi(4\pi + 3\sqrt{3})$	A1	**(8)**

5.	*(a)*	$2x - 3y - 3x\dfrac{dy}{dx} - 2y\dfrac{dy}{dx} = 0$	M1 A2	
		$\dfrac{dy}{dx} = \dfrac{2x-3y}{3x+2y}$	M1 A1	
	(b)	grad $= 5$	M1	
		$\therefore\; y + 2 = 5(x - 2) \qquad [\, y = 5x - 12 \,]$	M1 A1	**(8)**

6.	*(a)*	$= \left\| \dfrac{1\times6+5\times3+(-1)\times(-6)}{\sqrt{1+25+1}\times\sqrt{36+9+36}} \right\|$	M1 A1	
		$= \dfrac{27}{\sqrt{27}\times\sqrt{81}} = \dfrac{\sqrt{27}}{9} = \dfrac{3\sqrt{3}}{9} = \frac{1}{3}\sqrt{3}$	M1 A1	
	(b)	$\sin(\angle AOB) = \sqrt{1-(\frac{1}{3}\sqrt{3})^2} = \sqrt{\frac{2}{3}}$	M1 A1	
		area $= \frac{1}{2} \times 3\sqrt{3} \times 9 \times \sqrt{\frac{2}{3}} = \frac{27}{2}\sqrt{2}$	M1 A1	
	(c)	$= OA \times \sin(\angle AOB) = 3\sqrt{3} \times \sqrt{\frac{2}{3}} = 3\sqrt{2}$	M1 A1	**(10)**
7.	*(a)*	$\dfrac{dx}{dt} = 2t-1, \quad \dfrac{dy}{dt} = \dfrac{4\times(1-t)-4t\times(-1)}{(1-t)^2} = \dfrac{4}{(1-t)^2}$	B1 M1	
		$\dfrac{dy}{dx} = \dfrac{4}{(2t-1)(1-t)^2}$	M1 A1	
	(b)	$t=-1, \ x=2, \ y=-2, \ \text{grad} = -\frac{1}{3}$	M1	
		$\therefore \ y+2 = -\frac{1}{3}(x-2)$	M1	
		$3y+6 = -x+2$		
		$x+3y+4=0$	A1	
	(c)	$t(t-1) + 3\times\dfrac{4t}{1-t} + 4 = 0$	M1	
		$-t(t-1)^2 + 12t + 4(1-t) = 0$		
		$t^3 - 2t^2 - 7t - 4 = 0$	A1	
		$t=-1$ is a solution $\therefore (t+1)$ is a factor	M1	
		$(t+1)(t^2-3t-4) = 0$	M1	
		$(t+1)(t+1)(t-4) = 0$	A1	
		$t=-1$ (at P) or $t=4 \quad \therefore Q(12, -\frac{16}{3})$	M1 A1	**(14)**
8.	*(a)*	$\int \frac{1}{P}\, dP = \int k\, dt$	M1	
		$\ln\|P\| = kt + c$	A1	
		$t=0, \ P=300 \Rightarrow \ln 300 = c$	M1	
		$\ln\|P\| = kt + \ln 300$		
		$\ln\left\|\dfrac{P}{300}\right\| = kt, \quad \dfrac{P}{300} = e^{kt}, \quad P = 300e^{kt}$	M1 A1	
	(b)	$t=1, \ P=360 \Rightarrow 360 = 300e^{k}$	M1	
		$k = \ln\frac{6}{5} = 0.182$ (3sf)	A1	
	(c)	$P = 300e^{0.1823t}$		
		when $t=2, \ P=432$; when $t=3, \ P=518$	B1	
		model does not seem suitable as data diverges from predictions	B1	
	(d)	$\int \frac{1}{P}\, dP = \int (0.4 - 0.25\cos 0.5t)\, dt$	M1	
		$\ln\|P\| = 0.4t - 0.5\sin 0.5t + c$	A1	
		$t=0, \ P=300 \Rightarrow \ln 300 = c$		
		$\ln\left\|\dfrac{P}{300}\right\| = 0.4t - 0.5\sin 0.5t \quad [\, P = 300e^{0.4t - 0.5\sin 0.5t}\,]$	M1 A1	
	(e)	second model: $t = 1, 2, 3 \Rightarrow P = 352, 438, 605$	M1 A1	
		the second model seems more suitable as it fits the data better	B1	**(16)**
			Total	**(75)**