1. A curve has the equation

$$x^2 + 2xy^2 + y = 4.$$

Find an expression for $\frac{dy}{dx}$ in terms of x and y. **(6)**

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2. Use integration by parts to find

$$\int x^2 e^{-x} \, dx.$$ **(7)**

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3. The first four terms in the series expansion of $(1 + ax)^n$ in ascending powers of x are

$$1 - 4x + 24x^2 + kx^3,$$

where a, n and k are constants and $|ax| < 1$.

(a) Find the values of a and n. **(6)**

(b) Show that $k = -160$. **(2)**

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4. *(a)* Use the trapezium rule with two intervals of equal width to find an estimate for the value of the integral

$$\int_0^3 e^{\cos x} \, dx,$$

giving your answer to 3 significant figures. **(5)**

(b) Use the trapezium rule with four intervals of equal width to find another estimate for the value of the integral to 3 significant figures. **(2)**

(c) Given that the true value of the integral lies between the estimates made in parts *(a)* and *(b)*, comment on the shape of the curve $y = e^{\cos x}$ in the interval $0 \le x \le 3$ and explain your answer. **(2)**

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5. A straight road passes through villages at the points A and B with position vectors $(9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k})$ and $(4\mathbf{j} + \mathbf{k})$ respectively, relative to a fixed origin.

The road ends at a junction at the point C with another straight road which lies along the line with equation

$$\mathbf{r} = (2\mathbf{i} + 16\mathbf{j} - \mathbf{k}) + \mu(-5\mathbf{i} + 3\mathbf{j}),$$

where μ is a scalar parameter.

(a) Find the position vector of C. **(5)**

Given that 1 unit on each coordinate axis represents 200 metres,

(b) find the distance, in kilometres, from the village at A to the junction at C. **(4)**

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6. A small town had a population of 9000 in the year 2001.

In a model, it is assumed that the population of the town, P, at time t years after 2001 satisfies the differential equation

$$\frac{dP}{dt} = 0.05Pe^{-0.05t}.$$

(a) Show that, according to the model, the population of the town in 2011 will be 13 300 to 3 significant figures. **(7)**

(b) Find the value which the population of the town will approach in the long term, according to the model. **(3)**

Turn over

7.

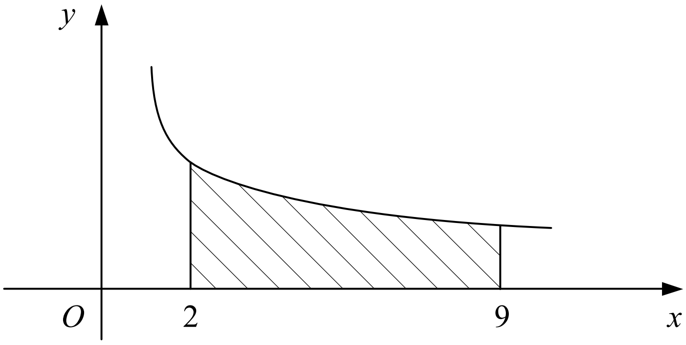

Figure 1

Figure 1 shows the curve with parametric equations

$$x = t^3 + 1, \quad y = \frac{2}{t}, \quad t > 0.$$

The shaded region is bounded by the curve, the x-axis and the lines $x = 2$ and $x = 9$.

(a) Find the area of the shaded region. **(5)**

(b) Show that the volume of the solid formed when the shaded region is rotated through 2π radians about the x-axis is 12π. **(3)**

(c) Find a cartesian equation for the curve in the form $y = \mathrm{f}(x)$. **(3)**

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8. *(a)* Show that the substitution $u = \sin x$ transforms the integral

$$\int \frac{6}{\cos x(2 - \sin x)} \, \mathrm{d}x$$

into the integral

$$\int \frac{6}{(1 - u^2)(2 - u)} \, \mathrm{d}u. \quad \textbf{(4)}$$

(b) Express $\dfrac{6}{(1 - u^2)(2 - u)}$ in partial fractions. **(4)**

(c) Hence, evaluate

$$\int_0^{\frac{\pi}{6}} \frac{6}{\cos x(2 - \sin x)} \, \mathrm{d}x,$$

giving your answer in the form $a \ln 2 + b \ln 3$, where a and b are integers. **(7)**

END

C4 Paper G – Marking Guide

1.	$2x + 2y^2 + 2x \times 2y\dfrac{dy}{dx} + \dfrac{dy}{dx} = 0$	M2 A2	
	$\dfrac{dy}{dx} = -\dfrac{2x+2y^2}{4xy+1}$	M1 A1	**(6)**
2.	$u = x^2,\ u' = 2x,\ v' = e^{-x},\ v = -e^{-x}$	M1 A1	
	$I = -x^2 e^{-x} - \int -2x e^{-x}\ dx = -x^2 e^{-x} + \int 2x e^{-x}\ dx$	A2	
	$u = 2x,\ u' = 2,\ v' = e^{-x},\ v = -e^{-x}$	M1	
	$I = -x^2 e^{-x} - 2x e^{-x} - \int -2e^{-x}\ dx$	A1	
	$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c$	A1	**(7)**
3.	*(a)* $(1 + ax)^n = 1 + nax + \frac{n(n-1)}{2}(ax)^2 + \ldots$	B1	
	$\therefore\ an = -4,\qquad \frac{a^2 n(n-1)}{2} = 24$	B1	
	$\Rightarrow a = \frac{-4}{n}$, sub. $\Rightarrow \frac{16}{n^2} \times \frac{n(n-1)}{2} = 24$	M1 A1	
	$8(n-1) = 24n,\qquad n = -\frac{1}{2},\ a = 8$	M1 A1	
	(b) $(1 + 8x)^{-\frac{1}{2}} = \ldots + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times 2}(8x)^3 + \ldots$	M1	
	$\therefore\ k = -\frac{5}{16} \times 512 = -160$	A1	**(8)**
4.	x: 0, 0.75, 1.5, 2.25, 3		
	y: 2.7183, 2.0786, 1.0733, 0.5336, 0.3716	B2	
	(a) $= \frac{1}{2} \times 1.5 \times [2.7183 + 0.3716 + 2(1.0733)] = 3.93$ (3sf)	B1 M1 A1	
	(b) $= \frac{1}{2} \times 0.75 \times [2.7183 + 0.3716 + 2(2.0786 + 1.0733 + 0.5336)]$	M1	
	$= 3.92$ (3sf)	A1	
	(c) curve must be above top of trapezia in some places and below in others hence position of ordinates determines whether estimate is high or low	B2	**(9)**
5.	*(a)* $\overrightarrow{AB} = (4\mathbf{j} + \mathbf{k}) - (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) = (-9\mathbf{i} + 12\mathbf{j} - \mathbf{k})$	M1	
	$\therefore\ \mathbf{r} = (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) + \lambda(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k})$	A1	
	at C, $2 - \lambda = -1,\ \lambda = 3$	M1 A1	
	$\therefore\ \overrightarrow{OC} = (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) + 3(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k}) = (-18\mathbf{i} + 28\mathbf{j} - \mathbf{k})$	A1	
	(b) $\overrightarrow{AC} = 3(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k}),\ AC = 3\sqrt{81+144+1} = 45.10$	M1 A1	
	$\therefore$ distance $= 200 \times 45.10 = 9020$ m $= 9.02$ km (3sf)	M1 A1	**(9)**

6.	*(a)*	$\int \frac{1}{P}\ \mathrm{d}P = \int 0.05\mathrm{e}^{-0.05t}\ \mathrm{d}t$	M1	
		$\ln\lvert P\rvert = -\mathrm{e}^{-0.05t} + c$	M1 A1	
		$t = 0,\ P = 9000 \Rightarrow \ln 9000 = -1 + c, \quad c = 1 + \ln 9000$	M1	
		$\ln\lvert P\rvert = 1 + \ln 9000 - \mathrm{e}^{-0.05t}$	A1	
		$t = 10 \Rightarrow \ln\lvert P\rvert = 1 + \ln 9000 - \mathrm{e}^{-0.5} = 9.498$	M1	
		$P = \mathrm{e}^{9.498} = 13339 = 13300$ (3sf)	A1	
	(b)	$t \to \infty,\ \ln\lvert P\rvert \to 1 + \ln 9000$	M1	
		$\therefore P \to \mathrm{e}^{1+\ln 9000} = 9000\mathrm{e} = 24465 = 24500$ (3sf)	M1 A1	**(10)**

7.	*(a)*	$x = 2 \Rightarrow t = 1, \quad x = 9 \Rightarrow t = 2$	B1	
		$\frac{\mathrm{d}x}{\mathrm{d}t} = 3t^2$	M1	
		$\therefore$ area $= \int_1^2 \frac{2}{t} \times 3t^2\ \mathrm{d}t = \int_1^2 6t\ \mathrm{d}t$	A1	
		$= [3t^2]_1^2 = 3(4 - 1) = 9$	M1 A1	
	(b)	$= \pi\int_1^2 (\frac{2}{t})^2 \times 3t^2\ \mathrm{d}t = \pi\int_1^2 12\ \mathrm{d}t$	M1	
		$= \pi[12t]_1^2 = 12\pi(2 - 1) = 12\pi$	M1 A1	
	(c)	$t = \frac{2}{y} \quad \therefore x = (\frac{2}{y})^3 + 1 = \frac{8}{y^3} + 1$	M1	
		$\therefore y^3 = \frac{8}{x-1}, \quad y = \frac{2}{\sqrt[3]{x-1}}$	M1 A1	**(11)**

8.	*(a)*	$u = \sin x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \cos x$	B1	
		$I = \int \frac{6\cos x}{\cos^2 x(2-\sin x)}\ \mathrm{d}x = \int \frac{6\cos x}{(1-\sin^2 x)(2-\sin x)}\ \mathrm{d}x$	M1	
		$= \int \frac{6}{(1-u^2)(2-u)}\ \mathrm{d}u$	M1 A1	
	(b)	$\frac{6}{(1+u)(1-u)(2-u)} \equiv \frac{A}{1+u} + \frac{B}{1-u} + \frac{C}{2-u}$		
		$6 \equiv A(1 - u)(2 - u) + B(1 + u)(2 - u) + C(1 + u)(1 - u)$	M1	
		$u = -1 \Rightarrow 6 = 6A \Rightarrow A = 1$	A1	
		$u = 1 \Rightarrow 6 = 2B \Rightarrow B = 3$	A1	
		$u = 2 \Rightarrow 6 = -3C \Rightarrow C = -2$	A1	
		$\therefore \frac{6}{(1-u^2)(2-u)} \equiv \frac{1}{1+u} + \frac{3}{1-u} - \frac{2}{2-u}$		
	(c)	$x = 0 \Rightarrow u = 0,\ x = \frac{\pi}{6} \Rightarrow u = \frac{1}{2}$	M1	
		$I = \int_0^{\frac{1}{2}} (\frac{1}{1+u} + \frac{3}{1-u} - \frac{2}{2-u})\ \mathrm{d}u$		
		$= [\ln\lvert 1 + u\rvert - 3\ln\lvert 1 - u\rvert + 2\ln\lvert 2 - u\rvert]_0^{\frac{1}{2}}$	M1 A2	
		$= (\ln\frac{3}{2} - 3\ln\frac{1}{2} + 2\ln\frac{3}{2}) - (0 + 0 + 2\ln 2)$	M1	
		$= 3\ln\frac{3}{2} + 3\ln 2 - 2\ln 2$		
		$= 3\ln 3 - 3\ln 2 + \ln 2 = 3\ln 3 - 2\ln 2$	M1 A1	**(15)**

Total **(75)**