**1.** Find

$$\int \cot^2 2x \;\mathrm{d}x. \qquad \textbf{(4)}$$

---

**2.** A curve has the equation

$$4 \cos x + 2 \sin y = 3.$$

*(a)* Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2 \sin x \sec y$. **(5)**

*(b)* Find an equation for the tangent to the curve at the point $(\frac{\pi}{3}, \frac{\pi}{6})$, giving your answer in the form $ax + by = c$, where $a$ and $b$ are integers. **(3)**

---

**3.** *(a)* Express $\dfrac{2+20x}{1+2x-8x^2}$ as a sum of partial fractions. **(4)**

*(b)* Hence find the series expansion of $\dfrac{2+20x}{1+2x-8x^2}$, $|x| < \frac{1}{4}$, in ascending powers of $x$ up to and including the term in $x^3$, simplifying each coefficient. **(5)**

---

**4.** The line $l_1$ passes through the points $P$ and $Q$ with position vectors $(-\mathbf{i} - 8\mathbf{j} + 3\mathbf{k})$ and $(2\mathbf{i} - 9\mathbf{j} + \mathbf{k})$ respectively, relative to a fixed origin.

*(a)* Find a vector equation for $l_1$. **(2)**

The line $l_2$ has the equation

$$\mathbf{r} = (6\mathbf{i} + a\mathbf{j} + b\mathbf{k}) + \mu(\mathbf{i} + 4\mathbf{j} - \mathbf{k})$$

and also passes through the point $Q$.

*(b)* Find the values of the constants $a$ and $b$. **(3)**

*(c)* Find, in degrees to 1 decimal place, the acute angle between lines $l_1$ and $l_2$. **(4)**

---

**5.** At time $t = 0$, a tank of height 2 metres is completely filled with water. Water then leaks from a hole in the side of the tank such that the depth of water in the tank, $y$ metres, after $t$ hours satisfies the differential equation

$$\frac{dy}{dt} = -ke^{-0.2t},$$

where $k$ is a positive constant,

*(a)* Find an expression for $y$ in terms of $k$ and $t$. **(4)**

Given that two hours after being filled the depth of water in the tank is 1.6 metres,

*(b)* find the value of $k$ to 4 significant figures. **(3)**

Given also that the hole in the tank is $h$ cm above the base of the tank,

*(c)* show that $h = 79$ to 2 significant figures. **(3)**

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**6.**

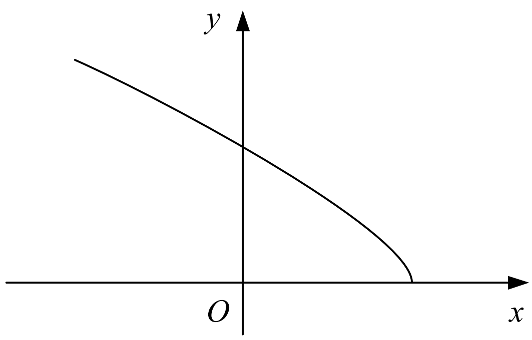


**Figure 1**

Figure 1 shows the curve with parametric equations

$$x = 2 - t^2, \quad y = t(t + 1), \quad t \geq 0.$$

*(a)* Find the coordinates of the points where the curve meets the coordinate axes. **(4)**

*(b)* Find the exact area of the region bounded by the curve and the coordinate axes. **(6)**

---

***Turn over***

7. *(a)* Prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(a^x) = a^x \ln a. \qquad \textbf{(3)}$$

A curve has the equation $y = 4^x - 2^{x-1} + 1$.

*(b)* Show that the tangent to the curve at the point where it crosses the $y$-axis has the equation

$$3x \ln 2 - 2y + 3 = 0. \qquad \textbf{(5)}$$

*(c)* Find the exact coordinates of the stationary point of the curve. **(4)**

---

8.

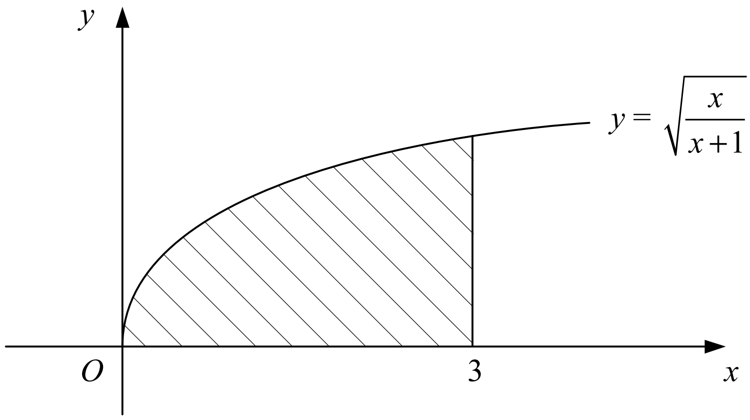


**Figure 2**

Figure 2 shows the curve with equation $y = \sqrt{\frac{x}{x+1}}$.

The shaded region is bounded by the curve, the $x$-axis and the line $x = 3$.

*(a)* *(i)* Use the trapezium rule with three strips to find an estimate for the area of the shaded region.

*(ii)* Use the trapezium rule with six strips to find an improved estimate for the area of the shaded region. **(7)**

The shaded region is rotated through $2\pi$ radians about the $x$-axis.

*(b)* Show that the volume of the solid formed is $\pi(3 - \ln 4)$. **(6)**

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**END**

# C4 Paper E – Marking Guide

| | | | |
|---|---|---|---|
| **1.** | $= \int (\text{cosec}^2\, 2x - 1)\ \text{d}x$ | M1 A1 | |
| | $= -\frac{1}{2}\cot 2x - x + c$ | M1 A1 | **(4)** |
| **2.** | *(a)* $-4\sin x + (2\cos y)\dfrac{\text{d}y}{\text{d}x} = 0$ | M1 A2 | |
| | $\dfrac{\text{d}y}{\text{d}x} = \dfrac{4\sin x}{2\cos y} = \dfrac{2\sin x}{\cos y} = 2\sin x \sec y$ | M1 A1 | |
| | *(b)* grad $= 2 \times \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = 2$ | B1 | |
| | $\therefore\ y - \frac{\pi}{6} = 2(x - \frac{\pi}{3})$ | M1 | |
| | $6y - \pi = 12x - 4\pi$ | | |
| | $4x - 2y = \pi$ | A1 | **(8)** |
| **3.** | *(a)* $\dfrac{2+20x}{1+2x-8x^2} = \dfrac{2+20x}{(1-2x)(1+4x)} \equiv \dfrac{A}{1-2x} + \dfrac{B}{1+4x}$ | B1 | |
| | $2 + 20x \equiv A(1 + 4x) + B(1 - 2x)$ | M1 | |
| | $x = \frac{1}{2} \quad \Rightarrow\ 12 = 3A \quad \Rightarrow\ A = 4$ | A1 | |
| | $x = -\frac{1}{4} \quad \Rightarrow\ -3 = \frac{3}{2}B \quad \Rightarrow\ B = -2 \qquad \dfrac{2+20x}{1+2x-8x^2} \equiv \dfrac{4}{1-2x} - \dfrac{2}{1+4x}$ | A1 | |
| | *(b)* $\dfrac{2+20x}{1+2x-8x^2} = 4(1-2x)^{-1} - 2(1+4x)^{-1}$ | | |
| | $(1-2x)^{-1} = 1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-2x)^3 + \ldots$ | M1 | |
| | $= 1 + 2x + 4x^2 + 8x^3 + \ldots$ | A1 | |
| | $(1+4x)^{-1} = 1 + (-1)(4x) + \frac{(-1)(-2)}{2}(4x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(4x)^3 + \ldots$ | | |
| | $= 1 - 4x + 16x^2 - 64x^3 + \ldots$ | A1 | |
| | $\dfrac{2+20x}{1+2x-8x^2} = 4(1 + 2x + 4x^2 + 8x^3 + \ldots) - 2(1 - 4x + 16x^2 - 64x^3 + \ldots)$ | M1 | |
| | $= 2 + 16x - 16x^2 - 160x^3 + \ldots$ | A1 | **(9)** |
| **4.** | *(a)* $\overrightarrow{PQ} = (2\mathbf{i} - 9\mathbf{j} + \mathbf{k}) - (-\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}) = (3\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ | M1 | |
| | $\therefore\ \mathbf{r} = (-\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}) + \lambda(3\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ | A1 | |
| | *(b)* $6 + \mu = 2 \quad \therefore\ \mu = -4$ | M1 | |
| | $a + 4\mu = -9 \quad \therefore\ a = 7$ | A1 | |
| | $b - \mu = 1 \quad \therefore\ b = -3$ | A1 | |
| | *(c)* $= \cos^{-1}\left\lvert \dfrac{3\times 1 + (-1)\times 4 + (-2)\times(-1)}{\sqrt{9+1+4}\times\sqrt{1+16+1}} \right\rvert$ | M1 A1 | |
| | $= \cos^{-1}\dfrac{1}{\sqrt{14}\times\sqrt{18}} = 86.4°$ (1dp) | M1 A1 | **(9)** |
| **5.** | *(a)* $\int \text{d}y = \int -k\text{e}^{-0.2t}\ \text{d}t$ | M1 | |
| | $y = 5k\text{e}^{-0.2t} + c$ | A1 | |
| | $t = 0,\ y = 2\ \Rightarrow\ 2 = 5k + c, \quad c = 2 - 5k$ | M1 | |
| | $\therefore\ y = 5k\text{e}^{-0.2t} - 5k + 2$ | A1 | |
| | *(b)* $t = 2,\ y = 1.6\ \Rightarrow\ 1.6 = 5k\text{e}^{-0.4} - 5k + 2$ | M1 | |
| | $k = \dfrac{-0.4}{5\text{e}^{-0.4} - 5} = 0.2427$ (4sf) | M1 A1 | |
| | *(c)* as $t \to \infty,\ y \to h$ (in metres) | M1 | |
| | $\therefore$ "$h$" $= -5k + 2 = 0.787$ m $= 78.7$ cm $\therefore\ h = 79$ | M1 A1 | **(10)** |

| | | | |
|---|---|---|---|
| **6.** | *(a)* | $x = 0 \Rightarrow t^2 = 2$ | |
| | | $t \geq 0 \quad \therefore\ t = \sqrt{2} \qquad \therefore\ (0, 2 + \sqrt{2})$ | M1 A1 |
| | | $y = 0 \Rightarrow t(t + 1) = 0$ | |
| | | $t \geq 0 \quad \therefore\ t = 0 \qquad \therefore\ (2, 0)$ | M1 A1 |
| | *(b)* | $\frac{\mathrm{d}x}{\mathrm{d}t} = -2t$ | M1 |
| | | area $= \int_{\sqrt{2}}^{0} t(t + 1) \times (-2t)\ \mathrm{d}t$ | A1 |
| | | $= \int_{0}^{\sqrt{2}} (2t^3 + 2t)\ \mathrm{d}t$ | |
| | | $= [\tfrac{1}{2}t^4 + \tfrac{2}{3}t^3]_0^{\sqrt{2}}$ | M1 A1 |
| | | $= (2 + \tfrac{4}{3}\sqrt{2}) - (0) = 2 + \tfrac{4}{3}\sqrt{2}$ | M1 A1 **(10)** |

---

| | | | |
|---|---|---|---|
| **7.** | *(a)* | let $y = a^x$, $\therefore\ \ln y = x \ln a$ | M1 |
| | | $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \ln a$ | M1 |
| | | $\frac{\mathrm{d}y}{\mathrm{d}x} = y \ln a = a^x \ln a \ \therefore\ \frac{\mathrm{d}}{\mathrm{d}x}(a^x) = a^x \ln a$ | A1 |
| | *(b)* | $\frac{\mathrm{d}y}{\mathrm{d}x} = 4^x \ln 4 - 2^{x-1} \ln 2$ | M1 A1 |
| | | $x = 0,\ y = \tfrac{3}{2}$, grad $= \ln 4 - \tfrac{1}{2}\ln 2 = \tfrac{3}{2}\ln 2$ | M1 |
| | | $\therefore\ y = (\tfrac{3}{2}\ln 2)x + \tfrac{3}{2}, \quad 2y = 3x \ln 2 + 3, \quad 3x \ln 2 - 2y + 3 = 0$ | M1 A1 |
| | *(c)* | $4^x \ln 4 - 2^{x-1} \ln 2 = 0$ | |
| | | $(2^x)^2 \times 2 \ln 2 - \tfrac{1}{2}(2^x) \ln 2 = 0$ | M1 |
| | | $\tfrac{1}{2}(2^x) \ln 2[4(2^x) - 1] = 0$ | M1 |
| | | $2^x = \tfrac{1}{4},\ x = -2 \ \therefore\ (-2, \tfrac{15}{16})$ | A2 **(12)** |

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**8.** *(a)*

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 0 | 0.5774 | 0.7071 | 0.7746 | 0.8165 | 0.8452 | 0.8660 |

B2

| | | |
|---|---|---|
| *(i)* | $\approx \tfrac{1}{2} \times 1 \times [0 + 0.8660 + 2(0.7071 + 0.8165)] = 1.96$ (3sf) | B1 M1 A1 |
| *(ii)* | $\approx \tfrac{1}{2}\times 0.5\times[0+0.8660+2(0.5774+0.7071+0.7746+0.8165+0.8452)]$ | |
| | $= 2.08$ (3sf) | M1 A1 |

| | | |
|---|---|---|
| *(b)* | $= \pi\int_0^3 \frac{x}{x+1}\ \mathrm{d}x$ | M1 |
| | $= \pi\int_0^3 \frac{x+1-1}{x+1}\ \mathrm{d}x = \pi\int_0^3 (1 - \frac{1}{x+1})\ \mathrm{d}x$ | M1 |
| | $= \pi[x - \ln\lvert x + 1\rvert]_0^3$ | M1 A1 |
| | $= \pi\{(3 - \ln 4) - (0)\} = \pi(3 - \ln 4)$ | M1 A1 **(13)** |

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Total **(75)**