1. *(a)* Find the binomial expansion of $(2 - 3x)^{-3}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(5)**

(b) State the set of values of x for which your expansion is valid. **(1)**

---

2. A curve has the equation

$$x^2 + 3xy - 2y^2 + 17 = 0.$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **(5)**

(b) Find an equation for the normal to the curve at the point $(3, -2)$. **(3)**

---

3. *(a)* Find the values of the constants A, B, C and D such that

$$\frac{2x^3 - 5x^2 + 6}{x^2 - 3x} \equiv Ax + B + \frac{C}{x} + \frac{D}{x-3}.$$ **(5)**

(b) Evaluate

$$\int_1^2 \frac{2x^3 - 5x^2 + 6}{x^2 - 3x}\ \mathrm{d}x,$$

giving your answer in the form $p + q \ln 2$, where p and q are integers. **(5)**

---

4. A mathematician is selling goods at a car boot sale. She believes that the rate at which she makes sales depends on the length of time since the start of the sale, t hours, and the total value of sales she has made up to that time, £x.

She uses the model

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k(5-t)}{x},$$

where k is a constant.

Given that after two hours she has made sales of £96 in total,

(a) solve the differential equation and show that she made £72 in the first hour of the sale. **(8)**

The mathematician believes that is it not worth staying at the sale once she is making sales at a rate of less than £10 per hour.

(b) Verify that at 3 hours and 5 minutes after the start of the sale, she should have already left. **(4)**

---

5. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$$

and $$\mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -6 \end{pmatrix} + t\begin{pmatrix} 3 \\ a \\ b \end{pmatrix},$$

where a and b are constants and s and t are scalar parameters.

Given that the two lines are perpendicular,

(a) find a linear relationship between a and b. **(2)**

Given also that the two lines intersect,

(b) find the values of a and b, **(8)**

(c) find the coordinates of the point where they intersect. **(2)**

Turn over

6.

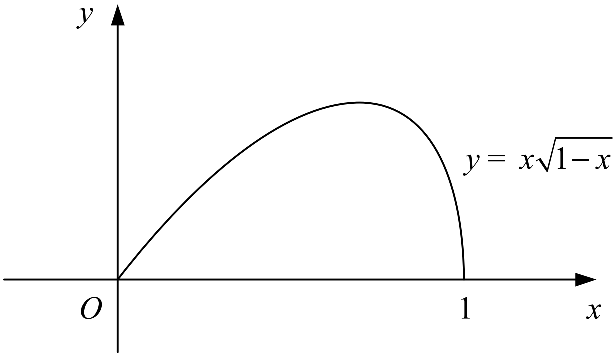

Figure 1

Figure 1 shows the curve with equation $y = x\sqrt{1-x}$, $0 \le x \le 1$.

(a) Use the substitution $u^2 = 1 - x$ to show that the area of the region bounded by the curve and the x-axis is $\frac{4}{15}$. **(8)**

(b) Find, in terms of π, the volume of the solid formed when the region bounded by the curve and the x-axis is rotated through 360° about the x-axis. **(5)**

---

7. A curve has parametric equations

$$x = 3\cos^2 t, \quad y = \sin 2t, \quad 0 \le t < \pi.$$

(a) Show that $\dfrac{dy}{dx} = -\frac{2}{3}\cot 2t$. **(4)**

(b) Find the coordinates of the points where the tangent to the curve is parallel to the x-axis. **(3)**

(c) Show that the tangent to the curve at the point where $t = \frac{\pi}{6}$ has the equation

$$2x + 3\sqrt{3}\,y = 9.$$ **(3)**

(d) Find a cartesian equation for the curve in the form $y^2 = f(x)$. **(4)**

END

C4 Paper D – Marking Guide

1.	*(a)*	$= 2^{-3}(1 - \frac{3}{2}x)^{-3} = \frac{1}{8}(1 - \frac{3}{2}x)^{-3}$	B1	
		$= \frac{1}{8}[1 + (-3)(-\frac{3}{2}x) + \frac{(-3)(-4)}{2}(-\frac{3}{2}x)^2 + \frac{(-3)(-4)(-5)}{3\times 2}(-\frac{3}{2}x)^3 + \ldots]$	M1	
		$= \frac{1}{8} + \frac{9}{16}x + \frac{27}{16}x^2 + \frac{135}{32}x^3 + \ldots$	A3	
	(b)	$\lvert x \rvert < \frac{2}{3}$	B1	**(6)**
2.	*(a)*	$2x + 3y + 3x\frac{dy}{dx} - 4y\frac{dy}{dx} = 0$	M1 A2	
		$\frac{dy}{dx} = \frac{2x+3y}{4y-3x}$	M1 A1	
	(b)	grad $= \frac{6-6}{-8-9} = 0$	M1	
		$\therefore$ normal parallel to y-axis $\therefore x = 3$	M1 A1	**(8)**
3.	*(a)*	$2x^3 - 5x^2 + 6 \equiv (Ax + B)x(x - 3) + C(x - 3) + Dx$	M1	
		$x = 0 \Rightarrow 6 = -3C \Rightarrow C = -2$		
		$x = 3 \Rightarrow 15 = 3D \Rightarrow D = 5$	A1	
		coeffs $x^3 \Rightarrow A = 2$	B1	
		coeffs $x^2 \Rightarrow -5 = B - 3A \Rightarrow B = 1$	M1 A1	
	(b)	$= \int_1^2 (2x + 1 - \frac{2}{x} + \frac{5}{x-3})\ dx$		
		$= [x^2 + x - 2\ln\lvert x\rvert + 5\ln\lvert x - 3\rvert]_1^2$	M1 A2	
		$= (4 + 2 - 2\ln 2 + 0) - (1 + 1 + 0 + 5\ln 2)$	M1	
		$= 4 - 7\ln 2$	A1	**(10)**
4.	*(a)*	$\int x\ dx = \int k(5 - t)\ dt$	M1	
		$\frac{1}{2}x^2 = k(5t - \frac{1}{2}t^2) + c$	M1 A1	
		$t = 0,\ x = 0 \Rightarrow c = 0$	B1	
		$t = 2,\ x = 96 \Rightarrow 4608 = 8k,\quad k = 576$	M1 A1	
		$t = 1 \Rightarrow \frac{1}{2}x^2 = 576 \times \frac{9}{2},\quad x = \sqrt{5184} = 72$	M1 A1	
	(b)	3 hours 5 mins $\Rightarrow t = 3.0833,\ x = \sqrt{12284} = 110.83$	M1 A1	
		$\therefore \frac{dx}{dt} = \frac{576(5 - 3.0833)}{110.83} = 9.96,\quad \frac{dx}{dt} < 10$ so she should have left	M1 A1	**(12)**

5.	*(a)*	$\begin{pmatrix}1\\4\\5\end{pmatrix}\cdot\begin{pmatrix}3\\a\\b\end{pmatrix} = 0 \quad \therefore\ 3 + 4a + 5b = 0$	M1 A1	
	(b)	$4 + s = -3 + 3t \quad (1)$		
		$1 + 4s = 1 + at \quad (2)$		
		$1 + 5s = -6 + bt \quad (3)$	B1	
		$(1) \Rightarrow s = 3t - 7$	M1	
		sub. (2) $\Rightarrow\ 1 + 4(3t - 7) = 1 + at$		
		$12t - 28 = at, \quad t(12 - a) = 28, \quad t = \frac{28}{12-a}$	M1 A1	
		sub. (3) $\Rightarrow\ 1 + 5(3t - 7) = -6 + bt$		
		$15t - 28 = bt, \quad t(15 - b) = 28, \quad t = \frac{28}{15-b}$	A1	
		$\frac{28}{12-a} = \frac{28}{15-b}, \quad 12 - a = 15 - b, \quad b = a + 3$	M1	
		sub *(a)* $\Rightarrow\ 3 + 4a + 5(a + 3) = 0, \ a = -2, b = 1$	M1 A1	
	(c)	$t = 2 \ \therefore\ \mathbf{r} = \begin{pmatrix}-3\\1\\-6\end{pmatrix} + 2\begin{pmatrix}3\\-2\\1\end{pmatrix} = \begin{pmatrix}3\\-3\\-4\end{pmatrix}, \ \therefore\ (3, -3, -4)$	M1 A1	**(12)**
6.	*(a)*	$u^2 = 1 - x \Rightarrow x = 1 - u^2, \ \frac{dx}{du} = -2u$	M1	
		$x = 0 \Rightarrow u = 1, \ x = 1 \Rightarrow u = 0$	B1	
		area $= \int_0^1 x\sqrt{1-x}\ dx = \int_1^0 (1 - u^2) \times u \times (-2u)\ du$	M1	
		$= \int_0^1 (2u^2 - 2u^4)\ du$	A1	
		$= [\frac{2}{3}u^3 - \frac{2}{5}u^5]_0^1$	M1 A1	
		$= (\frac{2}{3} - \frac{2}{5}) - (0) = \frac{4}{15}$	M1 A1	
	(b)	$= \pi\int_0^1 x^2(1 - x)\ dx$	M1	
		$= \pi\int_0^1 (x^2 - x^3)\ dx$		
		$= \pi[\frac{1}{3}x^3 - \frac{1}{4}x^4]_0^1$	M1 A1	
		$= \pi\{(\frac{1}{3} - \frac{1}{4}) - (0)\} = \frac{1}{12}\pi$	M1 A1	**(13)**
7.	*(a)*	$\frac{dx}{dt} = 6\cos t \times (-\sin t), \quad \frac{dy}{dt} = 2\cos 2t$	M1 A1	
		$\frac{dy}{dx} = \frac{2\cos 2t}{-6\cos t \sin t} = \frac{2\cos 2t}{-3\sin 2t} = -\frac{2}{3}\cot 2t$	M1 A1	
	(b)	$-\frac{2}{3}\cot 2t = 0 \Rightarrow 2t = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow t = \frac{\pi}{4}, \frac{3\pi}{4}$	M1 A1	
		$\therefore\ (\frac{3}{2}, 1), \ (\frac{3}{2}, -1)$	A1	
	(c)	$t = \frac{\pi}{6}, \ x = \frac{9}{4}, \ y = \frac{\sqrt{3}}{2}$, grad $= -\frac{2}{3\sqrt{3}}$	B1	
		$\therefore\ y - \frac{\sqrt{3}}{2} = -\frac{2}{3\sqrt{3}}(x - \frac{9}{4})$	M1	
		$6\sqrt{3}\,y - 9 = -4x + 9$		
		$2x + 3\sqrt{3}\,y = 9$	A1	
	(d)	$y^2 = \sin^2 2t = 4\sin^2 t\cos^2 t = 4(1 - \cos^2 t)\cos^2 t$	M2	
		$\cos^2 t = \frac{x}{3} \quad \therefore y^2 = 4(1 - \frac{x}{3})\frac{x}{3}, \quad y^2 = \frac{4}{9}x(3 - x)$	M1 A1	**(14)**

Total **(75)**