1. Use integration by parts to show that

$$\int_1^2 x \ln x \, dx = 2\ln 2 - \tfrac{3}{4}. \qquad \textbf{(6)}$$

---

2. *(a)* Use the trapezium rule with two intervals of equal width to find an approximate value for the integral

$$\int_0^2 \arctan x \, dx. \qquad \textbf{(5)}$$

(b) Use the trapezium rule with four intervals of equal width to find an improved approximation for the value of the integral. **(2)**

---

3. A curve has the equation

$$3x^2 - 2x + xy + y^2 - 11 = 0.$$

The point P on the curve has coordinates $(-1, 3)$.

(a) Show that the normal to the curve at P has the equation $y = 2 - x$. **(7)**

(b) Find the coordinates of the point where the normal to the curve at P meets the curve again. **(4)**

---

4. The points A and B have coordinates $(3, 9, -7)$ and $(13, -6, -2)$ respectively.

(a) Find, in vector form, an equation for the line l which passes through A and B. **(2)**

(b) Show that the point C with coordinates $(9, 0, -4)$ lies on l. **(2)**

The point D is the point on l closest to the origin, O.

(c) Find the coordinates of D. **(4)**

(d) Find the area of triangle OAB to 3 significant figures. **(3)**

---

5. A bath is filled with hot water which is allowed to cool. The temperature of the water is θ°C after cooling for t minutes and the temperature of the room is assumed to remain constant at 20°C.

Given that the rate at which the temperature of the water decreases is proportional to the difference in temperature between the water and the room,

(a) write down a differential equation connecting θ and t. **(2)**

Given also that the temperature of the water is initially 37°C and that it is 36°C after cooling for four minutes,

(b) find, to 3 significant figures, the temperature of the water after ten minutes. **(8)**

Advice suggests that the temperature of the water should be allowed to cool to 33°C before a child gets in.

(c) Find, to the nearest second, how long a child should wait before getting into the bath. **(3)**

Turn over

6.

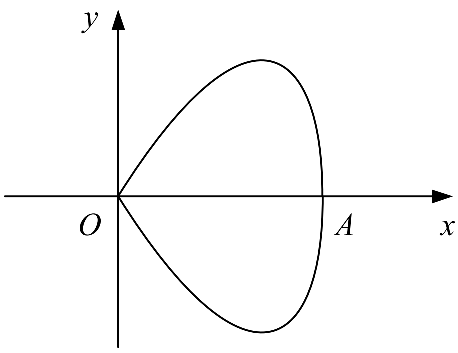

Figure 1

Figure 1 shows the curve with parametric equations

$$x = 3\sin t, \quad y = 2\sin 2t, \quad 0 \leq t < \pi.$$

The curve meets the x-axis at the origin, O, and at the point A.

(a) Find the value of t at O and the value of t at A. **(2)**

The region enclosed by the curve is rotated through π radians about the x-axis.

(b) Show that the volume of the solid formed is given by

$$\int_0^{\frac{\pi}{2}} 12\pi \sin^2 2t \cos t \;\mathrm{d}t.$$ **(3)**

(c) Using the substitution $u = \sin t$, or otherwise, evaluate this integral, giving your answer as an exact multiple of π. **(8)**

7.

$$\mathrm{f}(x) = \frac{8 - x}{(1 + x)(2 - x)}, \quad |x| < 1.$$

(a) Express $\mathrm{f}(x)$ in partial fractions. **(3)**

(b) Show that

$$\int_0^{\frac{1}{2}} \mathrm{f}(x)\;\mathrm{d}x = \ln k,$$

where k is an integer to be found. **(5)**

(c) Find the series expansion of $\mathrm{f}(x)$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(6)**

END

C4 Paper C – Marking Guide

1. $u = \ln x,\ u' = \frac{1}{x},\ v' = x,\ v = \frac{1}{2}x^2$ — M1

$I = [\frac{1}{2}x^2 \ln x]_1^2 - \int_1^2 \frac{1}{2}x \ \mathrm{d}x$ — A1

$= [\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2]_1^2$ — M1 A1

$= (2 \ln 2 - 1) - (0 - \frac{1}{4}) = 2 \ln 2 - \frac{3}{4}$ — M1 A1 **(6)**

2.

x	0	0.5	1	1.5	2	
$\arctan x$	0	0.4636	0.7854	0.9828	1.1071	B2

(a) $\approx \frac{1}{2} \times 1 \times [0 + 1.1071 + 2(0.7854)] = 1.34$ (3sf) — B1 M1 A1

(b) $\approx \frac{1}{2} \times 0.5 \times [0 + 1.1071 + 2(0.4636 + 0.7854 + 0.9828)] = 1.39$ (3sf) — M1 A1 **(7)**

---

3. *(a)* $6x - 2 + y + x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ — M1 A2

$(-1, 3) \Rightarrow -6 - 2 + 3 - \frac{\mathrm{d}y}{\mathrm{d}x} + 6\frac{\mathrm{d}y}{\mathrm{d}x} = 0,\quad \frac{\mathrm{d}y}{\mathrm{d}x} = 1$ — M1 A1

grad of normal $= -1$

$\therefore\ y - 3 = -(x + 1)$ — M1

$y = 2 - x$ — A1

(b) sub. $\Rightarrow\ 3x^2 - 2x + x(2 - x) + (2 - x)^2 - 11 = 0$ — M1

$3x^2 - 4x - 7 = 0$ — A1

$(3x - 7)(x + 1) = 0$ — M1

$x = -1$ (at P) or $\frac{7}{3}\ \ \therefore (\frac{7}{3}, -\frac{1}{3})$ — A1 **(11)**

---

4. *(a)* $\overrightarrow{AB} = \begin{pmatrix} 10 \\ -15 \\ 5 \end{pmatrix},\quad \therefore\ \mathbf{r} = \begin{pmatrix} 3 \\ 9 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ — M1 A1

(b) $3 + 2\lambda = 9\ \therefore\ \lambda = 3$ — M1

when $\lambda = 3,\ \mathbf{r} = \begin{pmatrix} 3 \\ 9 \\ -7 \end{pmatrix} + 3\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \\ -4 \end{pmatrix}\ \therefore (9, 0, -4)$ lies on l — A1

(c) $\overrightarrow{OD} = \begin{pmatrix} 3+2\lambda \\ 9-3\lambda \\ -7+\lambda \end{pmatrix}\ \therefore\ \begin{pmatrix} 3+2\lambda \\ 9-3\lambda \\ -7+\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 0$ — M1

$6 + 4\lambda - 27 + 9\lambda - 7 + \lambda = 0$ — A1

$\lambda = 2\ \therefore\ \overrightarrow{OD} = \begin{pmatrix} 7 \\ 3 \\ -5 \end{pmatrix},\ D\ (7, 3, -5)$ — M1 A1

(d) $AB = \sqrt{100 + 225 + 25} = \sqrt{350},\ OD = \sqrt{49 + 9 + 25} + \sqrt{83}$ — M1

area $= \frac{1}{2} \times \sqrt{350} \times \sqrt{83} = 85.2$ (3sf) — M1 A1 **(11)**

5.	*(a)*	$\dfrac{d\theta}{dt} = -k(\theta - 20)$	B2	
	(b)	$\int \frac{1}{\theta - 20}\ d\theta = \int -k\ dt$	M1	
		$\ln\lvert \theta - 20 \rvert = -kt + c$	M1 A1	
		$t = 0,\ \theta = 37 \ \Rightarrow\ c = \ln 17$	M1	
		$\ln\left\lvert \frac{\theta - 20}{17} \right\rvert = -kt, \quad \theta = 20 + 17e^{-kt}$	A1	
		$t = 4,\ \theta = 36 \ \Rightarrow \quad 36 = 20 + 17e^{-4k}$	M1	
		$k = -\frac{1}{4}\ln\frac{16}{17} = 0.01516$	A1	
		$t = 10,\ \theta = 20 + 17e^{-0.01516 \times 10} = 34.6°\text{C (3sf)}$	A1	
	(c)	$33 = 20 + 17e^{-0.01516t}$	M1	
		$t = -\frac{1}{0.01516}\ln\frac{13}{17} = 17.70$ minutes = 17 mins 42 secs	M1 A1	**(13)**
6.	*(a)*	$x = 0 \ \Rightarrow\ t = 0$ at O	B1	
		$y = 0 \ \Rightarrow\ t = 0$ (at O) or $\frac{\pi}{2}$ $\quad \therefore\ t = \frac{\pi}{2}$ at A	B1	
	(b)	= volume when region above x-axis is rotated through 2π		
		$\dfrac{dx}{dt} = 3\cos t$	M1	
		$\therefore$ volume $= \pi\int_0^{\frac{\pi}{2}} (2\sin 2t)^2 \times 3\cos t\ dt = \int_0^{\frac{\pi}{2}} 12\pi\sin^2 2t\cos t\ dt$	M1 A1	
	(c)	$t = 0 \ \Rightarrow\ u = 0,\ t = \frac{\pi}{2} \ \Rightarrow\ u = 1,\ \dfrac{du}{dt} = \cos t$	B1	
		$\sin^2 2t = 4\sin^2 t\cos^2 t = 4\sin^2 t\,(1 - \sin^2 t)$	M1	
		$\therefore\ = \int_0^1 12\pi \times 4u^2(1 - u^2)\ du$	M1	
		$= 48\pi\int_0^1 (u^2 - u^4)\ du$	A1	
		$= 48\pi[\frac{1}{3}u^3 - \frac{1}{5}u^5]_0^1$	M1 A1	
		$= 48\pi[(\frac{1}{3} - \frac{1}{5}) - (0)] = \frac{32}{5}\pi$	M1 A1	**(13)**
7.	*(a)*	$\dfrac{8 - x}{(1 + x)(2 - x)} \equiv \dfrac{A}{1 + x} + \dfrac{B}{2 - x}$		
		$8 - x \equiv A(2 - x) + B(1 + x)$	M1	
		$x = -1 \quad \Rightarrow \quad 9 = 3A \quad \Rightarrow \quad A = 3$	A1	
		$x = 2 \quad \Rightarrow \quad 6 = 3B \quad \Rightarrow \quad B = 2 \ \therefore\ f(x) = \dfrac{3}{1 + x} + \dfrac{2}{2 - x}$	A1	
	(b)	$= \int_0^{\frac{1}{2}} (\frac{3}{1 + x} + \frac{2}{2 - x})\ dx = [3\ln\lvert 1 + x \rvert - 2\ln\lvert 2 - x \rvert]_0^{\frac{1}{2}}$	M1 A1	
		$= (3\ln\frac{3}{2} - 2\ln\frac{3}{2}) - (0 - 2\ln 2)$	M1	
		$= \ln\frac{3}{2} + \ln 4 = \ln 6$	M1 A1	
	(c)	$f(x) = 3(1 + x)^{-1} + 2(2 - x)^{-1}$		
		$(1 + x)^{-1} = 1 - x + x^2 - x^3 + \ldots$	B1	
		$(2 - x)^{-1} = 2^{-1}(1 - \frac{1}{2}x)^{-1}$	M1	
		$= \frac{1}{2}[1 + (-1)(-\frac{1}{2}x) + \frac{(-1)(-2)}{2}(-\frac{1}{2}x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-\frac{1}{2}x)^3 + \ldots]$	M1	
		$= \frac{1}{2}(1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots)$	A1	
		$\therefore\ f(x) = 3(1 - x + x^2 - x^3 + \ldots) + (1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots)$	M1	
		$= 4 - \frac{5}{2}x + \frac{13}{4}x^2 - \frac{23}{8}x^3 + \ldots$	A1	**(14)**
			Total	**(75)**