1. Use integration by parts to find

$$\int x^2 \sin x \ \mathrm{d}x.$$ **(6)**

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2. Given that $y = -2$ when $x = 1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 \sqrt{x},$$

giving your answer in the form $y = \mathrm{f}(x)$. **(7)**

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3. A curve has the equation

$$4x^2 - 2xy - y^2 + 11 = 0.$$

Find an equation for the normal to the curve at the point with coordinates $(-1, -3)$. **(8)**

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4. *(a)* Expand $(1 + ax)^{-3}$, $|ax| < 1$, in ascending powers of $x$ up to and including the term in $x^3$. Give each coefficient as simply as possible in terms of the constant $a$. **(3)**

Given that the coefficient of $x^2$ in the expansion of $\frac{6 - x}{(1 + ax)^3}$, $|ax| < 1$, is 3,

*(b)* find the two possible values of $a$. **(4)**

Given also that $a < 0$,

*(c)* show that the coefficient of $x^3$ in the expansion of $\frac{6 - x}{(1 + ax)^3}$ is $\frac{14}{9}$. **(2)**

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**5.**

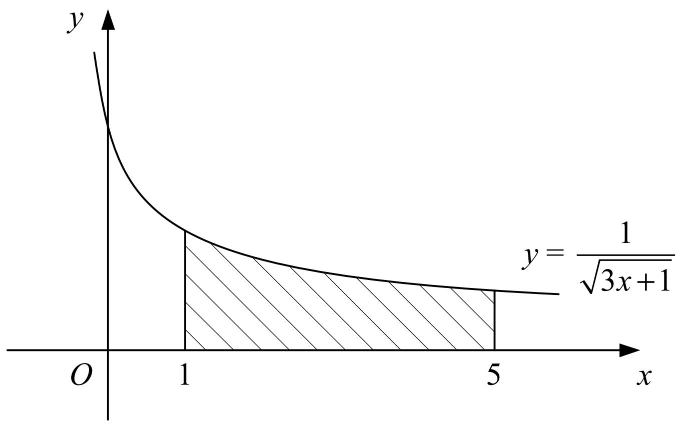


**Figure 1**

Figure 1 shows the curve with equation $y = \dfrac{1}{\sqrt{3x+1}}$.

The shaded region is bounded by the curve, the $x$-axis and the lines $x = 1$ and $x = 5$.

*(a)* Find the area of the shaded region. **(4)**

The shaded region is rotated completely about the $x$-axis.

*(b)* Find the volume of the solid formed, giving your answer in the form $k\pi \ln 2$, where $k$ is a simplified fraction. **(5)**

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**6.**

$$\text{f}(x) = \frac{15-17x}{(2+x)(1-3x)^2}, \quad x \neq -2, \quad x \neq \tfrac{1}{3}.$$

*(a)* Find the values of the constants $A$, $B$ and $C$ such that

$$\text{f}(x) = \frac{A}{2+x} + \frac{B}{1-3x} + \frac{C}{(1-3x)^2}.$$ **(4)**

*(b)* Find the value of

$$\int_{-1}^{0} \text{f}(x)\ \text{d}x,$$

giving your answer in the form $p + \ln q$, where $p$ and $q$ are integers. **(7)**

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***Turn over***

**7.**

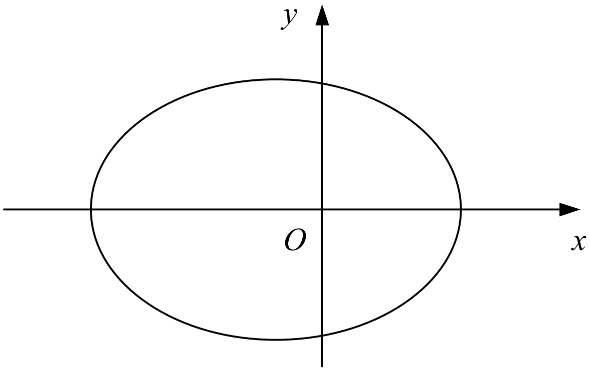


**Figure 2**

Figure 2 shows the curve with parametric equations

$$x = -1 + 4\cos\theta, \quad y = 2\sqrt{2}\sin\theta, \quad 0 \leq \theta < 2\pi.$$

The point $P$ on the curve has coordinates $(1, \sqrt{6})$.

*(a)* Find the value of $\theta$ at $P$. **(2)**

*(b)* Show that the normal to the curve at $P$ passes through the origin. **(7)**

*(c)* Find a cartesian equation for the curve. **(3)**

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**8.** The line $l_1$ passes through the points $A$ and $B$ with position vectors $(-3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ and $(7\mathbf{i} - \mathbf{j} + 12\mathbf{k})$ respectively, relative to a fixed origin.

*(a)* Find a vector equation for $l_1$. **(2)**

The line $l_2$ has the equation

$$\mathbf{r} = (5\mathbf{j} - 7\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}).$$

The point $C$ lies on $l_2$ and is such that $AC$ is perpendicular to $BC$.

*(b)* Show that one possible position vector for $C$ is $(\mathbf{i} + 3\mathbf{j})$ and find the other. **(8)**

Assuming that $C$ has position vector $(\mathbf{i} + 3\mathbf{j})$,

*(c)* find the area of triangle $ABC$, giving your answer in the form $k\sqrt{5}$. **(3)**

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**END**

## C4 Paper B – Marking Guide

| | | | |
|---|---|---|---|
| **1.** | $u = x^2,\ u' = 2x,\ v' = \sin x,\ v = -\cos x$ | M1 | |
| | $I = -x^2 \cos x - \int -2x \cos x \ \mathrm{d}x = -x^2 \cos x + \int 2x \cos x \ \mathrm{d}x$ | A2 | |
| | $u = 2x,\ u' = 2,\ v' = \cos x,\ v = \sin x$ | M1 | |
| | $I = -x^2 \cos x + 2x \sin x - \int 2 \sin x \ \mathrm{d}x$ | A1 | |
| | $= -x^2 \cos x + 2x \sin x + 2 \cos x + c$ | A1 | **(6)** |
| **2.** | $\int \frac{1}{y^2} \ \mathrm{d}y = \int \sqrt{x} \ \mathrm{d}x$ | M1 | |
| | $-y^{-1} = \frac{2}{3}x^{\frac{3}{2}} + c$ | M1 A1 | |
| | $x = 1,\ y = -2 \Rightarrow \frac{1}{2} = \frac{2}{3} + c, \quad c = -\frac{1}{6}$ | M1 A1 | |
| | $-\frac{1}{y} = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{6}, \quad \frac{1}{y} = \frac{1}{6} - \frac{2}{3}x^{\frac{3}{2}} = \frac{1}{6}(1 - 4x^{\frac{3}{2}})$ | M1 | |
| | $y = \frac{6}{1-4x^{\frac{3}{2}}}$ | A1 | **(7)** |
| **3.** | $8x - 2y - 2x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ | M1 A2 | |
| | $(-1, -3) \Rightarrow -8 + 6 + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 6\frac{\mathrm{d}y}{\mathrm{d}x} = 0, \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4}$ | M1 A1 | |
| | grad of normal $= -4$ | M1 | |
| | $\therefore\ y + 3 = -4(x + 1) \qquad [\ y = -4x - 7\ ]$ | M1 A1 | **(8)** |
| **4.** | *(a)* $= 1 + (-3)(ax) + \frac{(-3)(-4)}{2}(ax)^2 + \frac{(-3)(-4)(-5)}{3\times2}(ax)^3 + \ldots$ | M1 A1 | |
| | $= 1 - 3ax + 6a^2x^2 - 10a^3x^3 + \ldots$ | A1 | |
| | *(b)* $\frac{6-x}{(1+ax)^3} = (6 - x)(\ 1 - 3ax + 6a^2x^2 + \ldots)$ | | |
| | coeff. of $x^2 = 36a^2 + 3a = 3$ | M1 | |
| | $12a^2 + a - 1 = 0$ | A1 | |
| | $(4a - 1)(3a + 1) = 0$ | M1 | |
| | $a = -\frac{1}{3},\ \frac{1}{4}$ | A1 | |
| | *(c)* $a = -\frac{1}{3} \quad \therefore \frac{6-x}{(1+ax)^3} = (6 - x)(\ldots + \frac{2}{3}x^2 + \frac{10}{27}x^3 + \ldots)$ | M1 | |
| | coeff. of $x^3 = (6 \times \frac{10}{27}) + (-1 \times \frac{2}{3}) = \frac{20}{9} - \frac{2}{3} = \frac{14}{9}$ | A1 | **(9)** |
| **5.** | *(a)* $= \int_1^5 \frac{1}{\sqrt{3x+1}} \ \mathrm{d}x = [\frac{2}{3}(3x+1)^{\frac{1}{2}}]_1^5$ | M1 A1 | |
| | $= \frac{2}{3}(4 - 2) = \frac{4}{3}$ | M1 A1 | |
| | *(b)* $= \pi\int_1^5 \frac{1}{3x+1} \ \mathrm{d}x$ | M1 | |
| | $= \pi[\frac{1}{3}\ln\lvert 3x + 1\rvert]_1^5$ | M1 A1 | |
| | $= \frac{1}{3}\pi(\ln 16 - \ln 4) = \frac{1}{3}\pi \ln 4 = \frac{2}{3}\pi \ln 2 \qquad [\ k = \frac{2}{3}\ ]$ | M1 A1 | **(9)** |

| | | | |
|---|---|---|---|
| **6.** (a) | $15 - 17x \equiv A(1-3x)^2 + B(2+x)(1-3x) + C(2+x)$ | | |
| | $x = -2 \quad \Rightarrow \quad 49 = 49A \quad \Rightarrow \quad A = 1$ | B1 | |
| | $x = \frac{1}{3} \quad \Rightarrow \quad \frac{28}{3} = \frac{7}{3}C \quad \Rightarrow \quad C = 4$ | B1 | |
| | coeffs $x^2 \quad \Rightarrow \quad 0 = 9A - 3B \quad \Rightarrow \quad B = 3$ | M1 A1 | |
| (b) | $= \int_{-1}^{0} \left(\frac{1}{2+x} + \frac{3}{1-3x} + \frac{4}{(1-3x)^2}\right)\, dx$ | | |
| | $= [\ln\lvert 2+x\rvert - \ln\lvert 1-3x\rvert + \frac{4}{3}(1-3x)^{-1}]_{-1}^{0}$ | M1 A3 | |
| | $= (\ln 2 + 0 + \frac{4}{3}) - (0 - \ln 4 + \frac{1}{3})$ | M1 | |
| | $= 1 + \ln 8$ | M1 A1 | **(11)** |

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|---|---|---|---|
| **7.** (a) | $x = 1 \;\therefore\; -1 + 4\cos\theta = 1,\; \cos\theta = \frac{1}{2},\; \theta = \frac{\pi}{3}, \frac{5\pi}{3}$ | M1 | |
| | $y > 0 \;\therefore\; \sin\theta > 0 \;\therefore\; \theta = \frac{\pi}{3}$ | A1 | |
| (b) | $\frac{dx}{d\theta} = -4\sin\theta, \quad \frac{dy}{d\theta} = 2\sqrt{2}\cos\theta$ | M1 | |
| | $\therefore\; \frac{dy}{dx} = \frac{2\sqrt{2}\cos\theta}{-4\sin\theta}$ | M1 A1 | |
| | at $P$, grad $= -\frac{2\sqrt{2}\times\frac{1}{2}}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{2}}{2\sqrt{3}}$ | M1 | |
| | grad of normal $= \frac{2\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{6}$ | A1 | |
| | $\therefore\; y - \sqrt{6} = \sqrt{6}(x-1)$ | M1 | |
| | $y = \sqrt{6}\,x$, when $x = 0, y = 0 \;\therefore$ passes through origin | A1 | |
| (c) | $\cos\theta = \frac{x+1}{4},\; \sin\theta = \frac{y}{2\sqrt{2}}$ | M1 | |
| | $\therefore\; \frac{(x+1)^2}{16} + \frac{y^2}{8} = 1$ | M1 A1 | **(12)** |

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| | | | |
|---|---|---|---|
| **8.** (a) | $\overrightarrow{AB} = (7\mathbf{i} - \mathbf{j} + 12\mathbf{k}) - (-3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) = (10\mathbf{i} - 4\mathbf{j} + 10\mathbf{k})$ | M1 | |
| | $\therefore\; \mathbf{r} = (-3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + \lambda(5\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$ | A1 | |
| (b) | $\overrightarrow{OC} = [\mu\,\mathbf{i} + (5-2\mu)\mathbf{j} + (-7+7\mu)\mathbf{k}]$ | | |
| | $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = [(3+\mu)\mathbf{i} + (2-2\mu)\mathbf{j} + (-9+7\mu)\mathbf{k}]$ | M1 A1 | |
| | $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = [(-7+\mu)\mathbf{i} + (6-2\mu)\mathbf{j} + (-19+7\mu)\mathbf{k}]$ | A1 | |
| | $\overrightarrow{AC}\,.\,\overrightarrow{BC} = (3+\mu)(-7+\mu) + (2-2\mu)(6-2\mu) + (-9+7\mu)(-19+7\mu) = 0$ | M1 | |
| | $\mu^2 - 4\mu + 3 = 0$ | A1 | |
| | $(\mu - 1)(\mu - 3) = 0$ | M1 | |
| | $\mu = 1, 3 \quad \therefore\; \overrightarrow{OC} = (\mathbf{i} + 3\mathbf{j})$ or $(3\mathbf{i} - \mathbf{j} + 14\mathbf{k})$ | A2 | |
| (c) | $AC = \sqrt{16+0+4} = 2\sqrt{5},\; BC = \sqrt{36+16+144} = 14$ | M1 | |
| | area $= \frac{1}{2} \times 2\sqrt{5} \times 14 = 14\sqrt{5}$ | M1 A1 | **(13)** |

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|---|---|---|---|
| | | Total | **(75)** |