1. A curve has the equation

$$x^2(2 + y) - y^2 = 0.$$

Find an expression for $\frac{dy}{dx}$ in terms of x and y. **(6)**

2.
$$f(x) = \frac{3}{\sqrt{1-x}}, \quad |x| < 1.$$

(a) Show that $f(\frac{1}{10}) = \sqrt{10}$. **(2)**

(b) Expand f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(3)**

(c) Use your expansion to find an approximate value for $\sqrt{10}$, giving your answer to 8 significant figures. **(1)**

(d) Find, to 1 significant figure, the percentage error in your answer to part *(c)*. **(2)**

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3. Relative to a fixed origin, O, the line l has the equation

$$\mathbf{r} = (\mathbf{i} + p\mathbf{j} - 5\mathbf{k}) + \lambda(3\mathbf{i} - \mathbf{j} + q\mathbf{k}),$$

where p and q are constants and λ is a scalar parameter.

Given that the point A with coordinates $(-5, 9, -9)$ lies on l,

(a) find the values of p and q, **(3)**

(b) show that the point B with coordinates $(25, -1, 11)$ also lies on l. **(2)**

The point C lies on l and is such that OC is perpendicular to l.

(c) Find the coordinates of C. **(4)**

(d) Find the ratio $AC : CB$ **(2)**

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4. During a chemical reaction, a compound is being made from two other substances. At time t hours after the start of the reaction, x g of the compound has been produced. Assuming that $x = 0$ initially, and that

$$\frac{dx}{dt} = 2(x-6)(x-3),$$

(a) show that it takes approximately 7 minutes to produce 2 g of the compound. **(10)**

(b) Explain why it is not possible to produce 3 g of the compound. **(2)**

5.

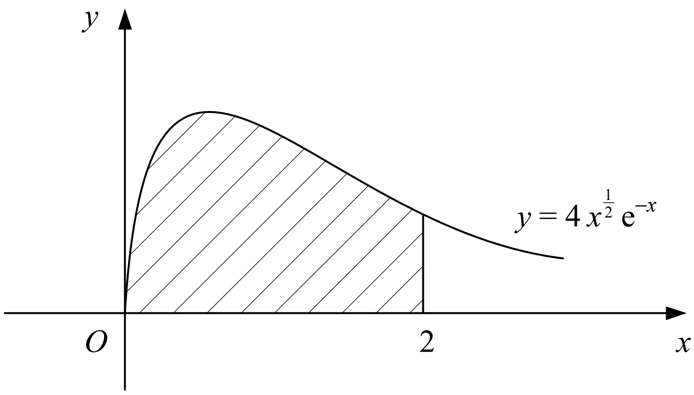

Figure 1

Figure 1 shows the curve with equation $y = 4x^{\frac{1}{2}}e^{-x}$.

The shaded region is bounded by the curve, the x-axis and the line $x = 2$.

(a) Use the trapezium rule with four intervals of equal width to estimate the area of the shaded region. **(5)**

The shaded region is rotated through 2π radians about the x-axis.

(b) Find, in terms of π and e, the exact volume of the solid formed. **(7)**

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6. *(a)* Find

$$\int 2\sin 3x \sin 2x \; dx.$$ **(4)**

(b) Use the substitution $u^2 = x + 1$ to evaluate

$$\int_0^3 \frac{x^2}{\sqrt{x+1}} \; dx.$$ **(8)**

Turn over

7.

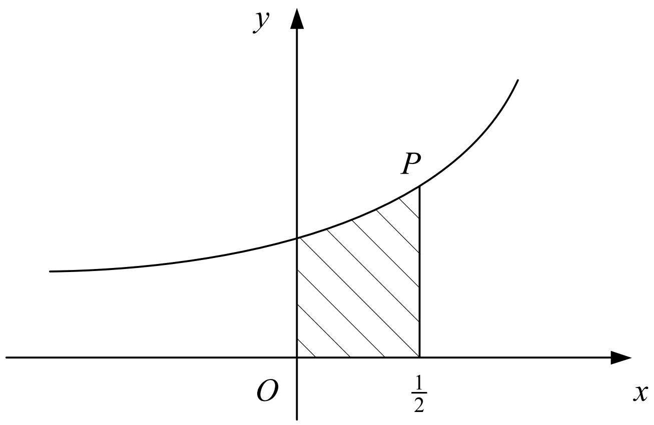

Figure 2

Figure 2 shows the curve with parametric equations

$$x = \cos 2t, \quad y = \operatorname{cosec} t, \quad 0 < t < \tfrac{\pi}{2}.$$

The point P on the curve has x-coordinate $\frac{1}{2}$.

(a) Find the value of the parameter t at P. **(2)**

(b) Show that the tangent to the curve at P has the equation

$$y = 2x + 1.$$ **(5)**

The shaded region is bounded by the curve, the coordinate axes and the line $x = \frac{1}{2}$.

(c) Show that the area of the shaded region is given by

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} k \cos t \, \mathrm{d}t,$$

where k is a positive integer to be found. **(4)**

(d) Hence find the exact area of the shaded region. **(3)**

END

C4 Paper A – Marking Guide

1.		$2x(2+y) + x^2\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	M2 A2	
		$\frac{dy}{dx} = \frac{2x(2+y)}{2y-x^2}$	M1 A1	**(6)**
2.	*(a)*	$f(\tfrac{1}{10}) = \frac{3}{\sqrt{1-\frac{1}{10}}} = \frac{3}{\sqrt{\frac{9}{10}}} = \frac{3}{(\frac{3}{\sqrt{10}})} = \sqrt{10}$	M1 A1	
	(b)	$= 3(1-x)^{-\frac{1}{2}} = 3[1 + (-\tfrac{1}{2})(-x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times 2}(-x)^3 + \ldots]$	M1	
		$= 3 + \tfrac{3}{2}x + \tfrac{9}{8}x^2 + \tfrac{15}{16}x^3 + \ldots$	A2	
	(c)	$\sqrt{10} = f(\tfrac{1}{10}) \approx 3 + \tfrac{3}{20} + \tfrac{9}{800} + \tfrac{15}{16000} = 3.1621875$ (8sf)	B1	
	(d)	$= \frac{\sqrt{10} - 3.1621875}{\sqrt{10}} \times 100\% = 0.003\%$ (1sf)	M1 A1	**(8)**
3.	*(a)*	$1 + 3\lambda = -5 \quad \therefore \lambda = -2$	M1	
		$p - \lambda = 9 \quad \therefore p = 7$	A1	
		$-5 + 2\lambda = 11 \quad \therefore q = 2$	A1	
	(b)	$1 + 3\lambda = 25 \quad \therefore \lambda = 8$	M1	
		when $\lambda = 8$, $\mathbf{r} = (\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}) + 8(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = (25\mathbf{i} - \mathbf{j} + 11\mathbf{k})$		
		$\therefore$ $(25, -1, 11)$ lies on l	A1	
	(c)	$\overrightarrow{OC} = [(1+3\lambda)\mathbf{i} + (7-\lambda)\mathbf{j} + (-5+2\lambda)\mathbf{k}]$		
		$\therefore$ $[(1+3\lambda)\mathbf{i} + (7-\lambda)\mathbf{j} + (-5+2\lambda)\mathbf{k}].(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = 0$	M1	
		$3 + 9\lambda - 7 + \lambda - 10 + 4\lambda = 0$	A1	
		$\lambda = 1 \quad \therefore \overrightarrow{OC} = (4\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})$, $C\,(4, 6, -3)$	M1 A1	
	(d)	$A : \lambda = -2$, $B : \lambda = 8$, $C : \lambda = 1 \quad \therefore AC : CB = 3 : 7$	M1 A1	**(11)**
4.	*(a)*	$\int \frac{1}{(x-6)(x-3)}\, dx = \int 2\, dt$	M1	
		$\frac{1}{(x-6)(x-3)} \equiv \frac{A}{x-6} + \frac{B}{x-3}$		
		$1 \equiv A(x-3) + B(x-6)$	M1	
		$x = 6 \Rightarrow A = \tfrac{1}{3}$, $x = 3 \Rightarrow B = -\tfrac{1}{3}$	A2	
		$\tfrac{1}{3}\int (\frac{1}{x-6} - \frac{1}{x-3})\, dx = \int 2\, dt$		
		$\ln\lvert x-6\rvert - \ln\lvert x-3\rvert = 6t + c$	M1 A1	
		$t = 0$, $x = 0 \ \therefore \ln 6 - \ln 3 = c$, $\quad c = \ln 2$	M1 A1	
		$x = 2 \Rightarrow \ln 4 - 0 = 6t + \ln 2$	M1	
		$t = \tfrac{1}{6}\ln 2 = 0.1155$ hrs $= 0.1155 \times 60$ mins $= 6.93$ mins ≈ 7 mins	A1	
	(b)	$\ln\left\lvert\frac{x-6}{2(x-3)}\right\rvert = 6t$, $\quad t = \tfrac{1}{6}\ln\left\lvert\frac{x-6}{2(x-3)}\right\rvert$		
		as $x \to 3$, $t \to \infty$ $\therefore$ cannot make 3 g	B2	**(12)**

5. *(a)*

x	0	0.5	1	1.5	2	
y	0	1.716	1.472	1.093	1.083	B2

area $\approx \frac{1}{2} \times 0.5 \times [0 + 1.083 + 2(1.716 + 1.472 + 1.093)] = 2.41$ (3sf) B1 M1 A1

(b) volume $= \pi \int_0^2 16x\, e^{-2x}\ dx$ M1

$u = 16x,\ u' = 16,\ v' = e^{-2x},\ v = -\frac{1}{2} e^{-2x}$ M1

$I = -8x\, e^{-2x} - \int -8e^{-2x}\ dx$ A2

$= -8x\, e^{-2x} - 4e^{-2x} + c$ A1

volume $= \pi[-8x\, e^{-2x} - 4e^{-2x}]_0^2$

$= \pi\{(-16e^{-4} - 4e^{-4}) - (0 - 4)\}$ M1

$= 4\pi(1 - 5e^{-4})$ A1 **(12)**

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6. *(a)* $= \int (\cos x - \cos 5x)\ dx$ M1 A1

$= \sin x - \frac{1}{5} \sin 5x + c$ M1 A1

(b) $u^2 = x + 1 \Rightarrow x = u^2 - 1,\ \frac{dx}{du} = 2u$ M1

$x = 0 \Rightarrow u = 1,\ x = 3 \Rightarrow u = 2$ B1

$I = \int_1^2 \frac{(u^2-1)^2}{u} \times 2u\ du = \int_1^2 (2u^4 - 4u^2 + 2)\ du$ M1 A1

$= [\frac{2}{5}u^5 - \frac{4}{3}u^3 + 2u]_1^2$ M1 A1

$= (\frac{64}{5} - \frac{32}{3} + 4) - (\frac{2}{5} - \frac{4}{3} + 2) = 5\frac{1}{15}$ M1 A1 **(12)**

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7. *(a)* $\cos 2t = \frac{1}{2},\ 2t = \frac{\pi}{3},\ t = \frac{\pi}{6}$ M1 A1

(b) $\frac{dx}{dt} = -2 \sin 2t,\ \frac{dy}{dt} = -\text{cosec } t \cot t$ M1

$\frac{dy}{dx} = \frac{-\text{cosec}\, t \cot t}{-2 \sin 2t}$ M1 A1

$t = \frac{\pi}{6},\ y = 2,$ grad $= 2$

$\therefore\ y - 2 = 2(x - \frac{1}{2})$ M1

$y = 2x + 1$ A1

(c) $x = 0 \Rightarrow t = \frac{\pi}{4}$ B1

$\therefore$ area $= \int_{\frac{\pi}{4}}^{\frac{\pi}{6}} \text{cosec } t \times (-2 \sin 2t)\ dt$ M1

$= -\int_{\frac{\pi}{4}}^{\frac{\pi}{6}} \text{cosec } t \times 4 \sin t \cos t\ dt = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 4 \cos t\ dt$ M1 A1

(d) $= [4 \sin t]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = 2\sqrt{2} - 2 = 2(\sqrt{2} - 1)$ M2 A1 **(14)**

Total **(75)**