1. $$\mathrm{f}(x) \equiv \frac{2x-3}{x-2}, \quad x \in \mathbb{R}, \quad x > 2.$$

*(a)* Find the range of f. **(2)**

*(b)* Show that $\mathrm{ff}(x) = x$ for all $x > 2$. **(3)**

*(c)* Hence, write down an expression for $\mathrm{f}^{-1}(x)$. **(1)**

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2. Solve each equation, giving your answers in exact form.

*(a)* $e^{4x-3} = 2$ **(3)**

*(b)* $\ln(2y - 1) = 1 + \ln(3 - y)$ **(4)**

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3. The curve $C$ has the equation $y = 2e^x - 6\ln x$ and passes through the point $P$ with $x$-coordinate 1.

*(a)* Find an equation for the tangent to $C$ at $P$. **(4)**

The tangent to $C$ at $P$ meets the coordinate axes at the points $Q$ and $R$.

*(b)* Show that the area of triangle $OQR$, where $O$ is the origin, is $\dfrac{9}{3-e}$. **(4)**

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4. *(a)* Express

$$\frac{x-10}{(x-3)(x+4)} - \frac{x-8}{(x-3)(2x-1)}$$

as a single fraction in its simplest form. **(5)**

*(b)* Hence, show that the equation

$$\frac{x-10}{(x-3)(x+4)} - \frac{x-8}{(x-3)(2x-1)} = 1$$

has no real roots. **(4)**

---

5. Find the values of $x$ in the interval $-180 < x < 180$ for which

$$\tan (x + 45)° - \tan x° = 4,$$

giving your answers to 1 decimal place. **(9)**

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6. *(a)* Sketch on the same diagram the graphs of $y = |x| - a$ and $y = |3x + 5a|$, where $a$ is a positive constant.

Show on your diagram the coordinates of any points where each graph meets the coordinate axes. **(6)**

*(b)* Solve the equation

$$|x| - a = |3x + 5a|.$$ **(4)**

---

7. *(a)* Use the identity

$$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$$

to prove that

$$\cos x \equiv 1 - 2 \sin^2 \tfrac{x}{2}.$$ **(3)**

*(b)* Prove that, for $\sin x \neq 0$,

$$\frac{1-\cos x}{\sin x} \equiv \tan \tfrac{x}{2}.$$ **(3)**

*(c)* Find the values of $x$ in the interval $0 \leq x \leq 360°$ for which

$$\frac{1-\cos x}{\sin x} = 2 \sec^2 \tfrac{x}{2} - 5,$$

giving your answers to 1 decimal place where appropriate. **(6)**

---

***Turn over***

**8.** A curve has the equation $y = (2x + 3)\,e^{-x}$.

*(a)* Find the exact coordinates of the stationary point of the curve. **(4)**

The curve crosses the $y$-axis at the point $P$.

*(b)* Find an equation for the normal to the curve at $P$. **(2)**

The normal to the curve at $P$ meets the curve again at $Q$.

*(c)* Show that the $x$-coordinate of $Q$ lies in the interval $[-2, -1]$. **(3)**

*(d)* Use the iterative formula

$$x_{n+1} = \frac{3 - 3e^{x_n}}{e^{x_n} - 2},$$

with $x_0 = -1$, to find $x_1$, $x_2$, $x_3$ and $x_4$. Give the value of $x_4$ to 2 decimal places. **(3)**

*(e)* Show that your value for $x_4$ is the $x$-coordinate of $Q$ correct to 2 decimal places. **(2)**

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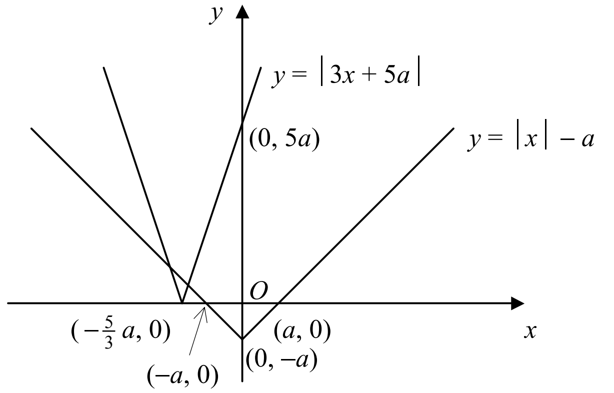
**END**

# C3 Paper L – Marking Guide

| | | | | |
|---|---|---|---|---|
| **1.** | *(a)* | $f(x) = \frac{2(x-2)+1}{x-2} = 2 + \frac{1}{x-2}$ | M1 | |
| | | $x > 2 \;\therefore\; f(x) > 2$ | A1 | |
| | *(b)* | $ff(x) = f(\frac{2x-3}{x-2}) = \frac{2(\frac{2x-3}{x-2})-3}{\frac{2x-3}{x-2}-2}$ | M1 | |
| | | $= \frac{2(2x-3)-3(x-2)}{(2x-3)-2(x-2)} = \frac{4x-6-3x+6}{2x-3-2x+4} = x$ | M1 A1 | |
| | *(c)* | $f^{-1}(x) = \frac{2x-3}{x-2}$ | B1 | **(6)** |
| **2.** | *(a)* | $4x - 3 = \ln 2$ | M1 | |
| | | $x = \frac{1}{4}(3 + \ln 2)$ | M1 A1 | |
| | *(b)* | $\ln(2y-1) - \ln(3-y) = \ln\frac{2y-1}{3-y} = 1$ | M1 | |
| | | $\frac{2y-1}{3-y} = e$ | A1 | |
| | | $2y - 1 = e(3-y), \quad y(e+2) = 3e + 1$ | M1 | |
| | | $y = \frac{3e+1}{e+2}$ | A1 | **(7)** |
| **3.** | *(a)* | $\frac{dy}{dx} = 2e^x - \frac{6}{x}$ | M1 | |
| | | $x = 1,\; y = 2e,\;$ grad $= 2e - 6$ | A1 | |
| | | $\therefore\; y - 2e = (2e-6)(x-1) \qquad [\, y = (2e-6)x + 6 \,]$ | M1 A1 | |
| | *(b)* | $x = 0 \Rightarrow y = 6$ | | |
| | | $y = 0 \Rightarrow (2e-6)x + 6 = 0$ | | |
| | | $x = \frac{-6}{2e-6} = \frac{3}{3-e}$ | M1 A1 | |
| | | area $= \frac{1}{2} \times 6 \times \frac{3}{3-e} = \frac{9}{3-e}$ | M1 A1 | **(8)** |
| **4.** | *(a)* | $= \frac{(x-10)(2x-1)-(x-8)(x+4)}{(x-3)(x+4)(2x-1)}$ | M1 A1 | |
| | | $= \frac{x^2-17x+42}{(x-3)(x+4)(2x-1)}$ | A1 | |
| | | $= \frac{(x-14)(x-3)}{(x-3)(x+4)(2x-1)} = \frac{x-14}{(x+4)(2x-1)}$ | M1 A1 | |
| | *(b)* | $\frac{x-14}{(x+4)(2x-1)} = 1, \quad x - 14 = 2x^2 + 7x - 4$ | M1 | |
| | | $x^2 + 3x + 5 = 0$ | A1 | |
| | | $b^2 - 4ac = 9 - 20 = -11$ | M1 | |
| | | $b^2 - 4ac < 0 \;\therefore$ no real roots | A1 | **(9)** |
| **5.** | | $\frac{\tan x + \tan 45}{1 - \tan x \tan 45} - \tan x = 4$ | M1 | |
| | | $\frac{\tan x + 1}{1 - \tan x} = 4 + \tan x$ | A1 | |
| | | $\tan x + 1 = (4 + \tan x)(1 - \tan x)$ | M1 | |
| | | $\tan^2 x + 4\tan x - 3 = 0$ | A1 | |
| | | $\tan x = \frac{-4 \pm \sqrt{16+12}}{2} = -2 \pm \sqrt{7}$ | M1 | |
| | | $x = 180 - 77.9, -77.9$ or $32.9, -180 + 32.9$ | B1 M1 | |
| | | $x = -147.1, -77.9, 32.9, 102.1$ (1dp) | A2 | **(9)** |

**6.** *(a)*




B3
B3

*(b)* $-x - a = 3x + 5a \quad \Rightarrow \quad x = -\frac{3}{2}a$ M1 A1

$-x - a = -(3x + 5a) \quad \Rightarrow \quad x = -2a, \qquad x = -2a, \ -\frac{3}{2}a$ M1 A1 **(10)**

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**7.** *(a)* $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$

let $A = B = \frac{x}{2}$ $\quad \cos x \equiv \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$ M1

$\cos x \equiv (1 - \sin^2 \frac{x}{2}) - \sin^2 \frac{x}{2}$ M1

$\cos x \equiv 1 - 2\sin^2 \frac{x}{2}$ A1

*(b)* $\text{LHS} \equiv \dfrac{1-(1-2\sin^2 \frac{x}{2})}{2\sin\frac{x}{2}\cos\frac{x}{2}}$ M1

$\equiv \dfrac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} \equiv \dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}} \equiv \tan\frac{x}{2} \equiv \text{RHS}$ M1 A1

*(c)* $\tan\frac{x}{2} = 2\sec^2\frac{x}{2} - 5, \qquad \tan\frac{x}{2} = 2(1 + \tan^2\frac{x}{2}) - 5$ M1

$2\tan^2\frac{x}{2} - \tan\frac{x}{2} - 3 = 0, \quad (2\tan\frac{x}{2} - 3)(\tan\frac{x}{2} + 1) = 0$ M1

$\tan\frac{x}{2} = -1 \text{ or } \frac{3}{2}$ A1

$\frac{x}{2} = 135 \text{ or } 56.310$ B1

$x = 112.6°$ (1dp), $270°$ A2 **(12)**

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**8.** *(a)* $\dfrac{dy}{dx} = 2 \times e^{-x} + (2x + 3) \times (-e^{-x}) = -(2x + 1)e^{-x}$ M1 A1

SP: $\quad -(2x + 1)e^{-x} = 0$

$x = -\frac{1}{2} \quad \therefore (-\frac{1}{2}, 2e^{\frac{1}{2}})$ M1 A1

*(b)* $x = 0, \ y = 3, \ \text{grad} = -1, \ \text{grad of normal} = 1$ M1

$\therefore \ y = x + 3$ A1

*(c)* $x + 3 = (2x + 3)e^{-x}$

$x + 3 - (2x + 3)e^{-x} = 0$ M1

let $f(x) = x + 3 - (2x + 3)e^{-x}$

$f(-2) = 8.4, \ f(-1) = -0.72$ M1

sign change, $f(x)$ continuous $\therefore$ root A1

*(d)* $x_1 = -1.1619, x_2 = -1.2218, x_3 = -1.2408, x_4 = -1.2465 = -1.25$ (2dp) M1 A2

*(e)* $f(-1.255) = 0.026, \ f(-1.245) = -0.016$ M1

sign change, $f(x)$ continuous $\therefore$ root A1 **(14)**

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Total **(75)**