1. *(a)* Find the exact value of x such that

$$3 \arctan (x - 2) + \pi = 0.$$ **(3)**

(b) Solve, for $-\pi < \theta < \pi$, the equation

$$\cos 2\theta - \sin \theta - 1 = 0,$$

giving your answers in terms of π. **(5)**

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2. *(a)* Express

$$\frac{4x}{x^2 - 9} - \frac{2}{x + 3}$$

as a single fraction in its simplest form. **(4)**

(b) Simplify

$$\frac{x^3 - 8}{3x^2 - 8x + 4}.$$ **(5)**

---

3. Differentiate each of the following with respect to x and simplify your answers.

(a) $\cot x^2$ **(2)**

(b) $x^2 \mathrm{e}^{-x}$ **(3)**

(c) $\dfrac{\sin x}{3 + 2\cos x}$ **(4)**

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4. *(a)* Find, as natural logarithms, the solutions of the equation

$$\mathrm{e}^{2x} - 8\mathrm{e}^{x} + 15 = 0.$$ **(4)**

(b) Use proof by contradiction to prove that $\log_2 3$ is irrational. **(6)**

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5. The function f is defined by

$$\mathrm{f} : x \to 3\mathrm{e}^{x-1}, \quad x \in \mathbb{R}.$$

(a) State the range of f. **(1)**

(b) Find an expression for $\mathrm{f}^{-1}(x)$ and state its domain. **(4)**

The function g is defined by

$$\mathrm{g} : x \to 5x - 2, \quad x \in \mathbb{R}.$$

Find, in terms of e,

(c) the value of $\mathrm{gf}(\ln 2)$, **(3)**

(d) the solution of the equation

$$\mathrm{f}^{-1}\mathrm{g}(x) = 4.$$ **(4)**

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6. $$\mathrm{f}(x) = 2x^2 + 3\ln(2 - x), \quad x \in \mathbb{R}, \quad x < 2.$$

(a) Show that the equation $\mathrm{f}(x) = 0$ can be written in the form

$$x = 2 - \mathrm{e}^{kx^2},$$

where k is a constant to be found. **(3)**

The root, α, of the equation $\mathrm{f}(x) = 0$ is 1.9 correct to 1 decimal place.

(b) Use the iteration formula

$$x_{n+1} = 2 - \mathrm{e}^{kx_n^2},$$

with $x_0 = 1.9$ and your value of k, to find α to 3 decimal places and justify the accuracy of your answer. **(5)**

(c) Solve the equation $\mathrm{f}'(x) = 0$. **(5)**

Turn over

7.

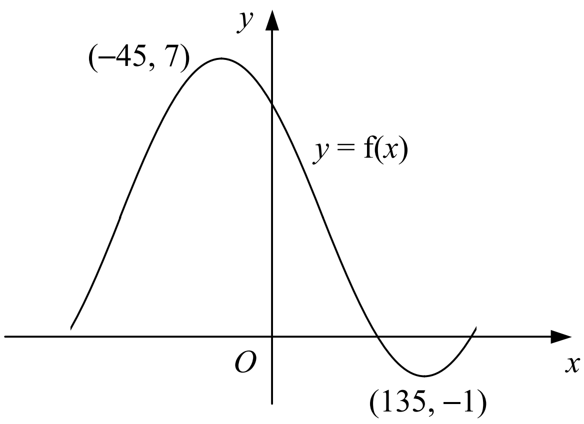


Figure 1

Figure 1 shows the curve $y = \mathrm{f}(x)$ which has a maximum point at (–45, 7) and a minimum point at (135, –1).

(a) Showing the coordinates of any stationary points, sketch on separate diagrams the graphs of

(i) $y = \mathrm{f}(|x|)$,

(ii) $y = 1 + 2\mathrm{f}(x)$. **(6)**

Given that

$$\mathrm{f}(x) = A + 2\sqrt{2}\cos x° - 2\sqrt{2}\sin x°, \quad x \in \mathbb{R}, \quad -180 \le x \le 180,$$

where A is a constant,

(b) show that f(x) can be expressed in the form

$$\mathrm{f}(x) = A + R\cos(x + \alpha)°,$$

where $R > 0$ and $0 < \alpha < 90$, **(3)**

(c) state the value of A, **(1)**

(d) find, to 1 decimal place, the x-coordinates of the points where the curve $y = \mathrm{f}(x)$ crosses the x-axis. **(4)**

END

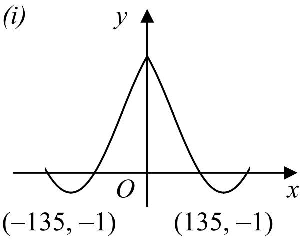
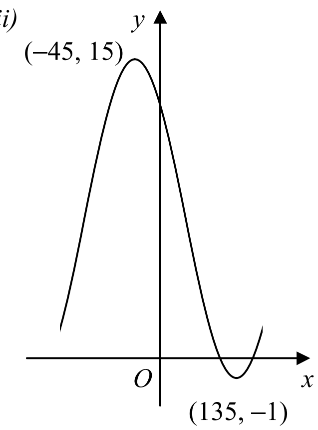
C3 Paper K – Marking Guide

1.	*(a)*	$\arctan(x-2) = -\frac{\pi}{3}$	M1	
		$x - 2 = \tan(-\frac{\pi}{3}) = -\sqrt{3}$	M1	
		$x = 2 - \sqrt{3}$	A1	
	(b)	$1 - 2\sin^2\theta - \sin\theta - 1 = 0$	M1	
		$2\sin^2\theta + \sin\theta = 0$		
		$\sin\theta(2\sin\theta + 1) = 0$	M1	
		$\sin\theta = 0$ or $-\frac{1}{2}$	A1	
		$\theta = 0$ or $-\frac{\pi}{6}, -\pi + \frac{\pi}{6}$		
		$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, 0$	A2	**(8)**

2.	*(a)*	$= \frac{4x}{(x+3)(x-3)} - \frac{2}{x+3}$	M1	
		$= \frac{4x-2(x-3)}{(x+3)(x-3)}$	M1	
		$= \frac{2x+6}{(x+3)(x-3)} = \frac{2(x+3)}{(x+3)(x-3)}$	M1	
		$= \frac{2}{x-3}$	A1	
	(b)	$2^3 - 8 = 0 \ \therefore \ (x-2)$ is a factor of $(x^3 - 8)$	B1	
		$$\begin{array}{r|rrrr} & x^2 & +\ 2x & +\ 4 & \\ \hline x-2 & x^3 & +\ 0x^2 & +\ 0x & -\ 8 \\ & x^3 & -\ 2x^2 & & \\ \hline & & 2x^2 & +\ 0x & \\ & & 2x^2 & -\ 4x & \\ \hline & & & 4x & -\ 8 \\ & & & 4x & -\ 8 \end{array}$$	M1 A1	
		$\therefore \ x^3 - 8 = (x-2)(x^2 + 2x + 4)$		
		$\therefore \ \frac{x^3-8}{3x^2-8x+4} = \frac{(x-2)(x^2+2x+4)}{(3x-2)(x-2)} = \frac{x^2+2x+4}{3x-2}$	M1 A1	**(9)**

3.	*(a)*	$= -\text{cosec}^2\, x^2 \times 2x = -2x\,\text{cosec}^2\, x^2$	M1 A1	
	(b)	$= 2x \times e^{-x} + x^2 \times (-e^{-x}) = xe^{-x}(2 - x)$	M1 A2	
	(c)	$= \frac{\cos x \times (3+2\cos x) - \sin x \times (-2\sin x)}{(3+2\cos x)^2}$	M1 A1	
		$= \frac{3\cos x + 2\cos^2 x + 2\sin^2 x}{(3+2\cos x)^2} = \frac{3\cos x + 2}{(3+2\cos x)^2}$	M1 A1	**(9)**

4.	*(a)*	$(e^x - 3)(e^x - 5) = 0$	M1 A1	
		$e^x = 3, 5$		
		$x = \ln 3, \ln 5$	M1 A1	
	(b)	assume $\log_2 3$ is rational	B1	
		$\therefore \ \log_2 3 = \frac{p}{q}$ where p and q are integers and $q \neq 0$	M1	
		$\Rightarrow \ 2^{\frac{p}{q}} = 3$	M1	
		$\Rightarrow \ 2^p = 3^q$	A1	
		2 and 3 are co-prime $\therefore$ only solution is $p = q = 0$	M1	
		but $q \neq 0 \ \therefore$ contradiction $\therefore \ \log_2 3$ is irrational	A1	**(10)**

5.	*(a)*	$f(x) > 0$	B1	
	(b)	$y = 3e^{x-1}, \quad x - 1 = \ln \frac{y}{3}$	M1	
		$x = 1 + \ln \frac{y}{3}$		
		$f^{-1}(x) = 1 + \ln \frac{x}{3}, \; x \in \mathbb{R}, \; x > 0$	M1 A2	
	(c)	$f(\ln 2) = 3e^{\ln 2 - 1} = 3e^{-1}e^{\ln 2} = 6e^{-1}$	M1 A1	
		$gf(\ln 2) = g(6e^{-1}) = 30e^{-1} - 2$	A1	
	(d)	$f^{-1}g(x) = f^{-1}(5x - 2) = 1 + \ln \frac{5x-2}{3}$	M1 A1	
		$\therefore \; 1 + \ln \frac{5x-2}{3} = 4, \quad \frac{5x-2}{3} = e^3$	M1	
		$x = \frac{1}{5}(3e^3 + 2)$	A1	**(12)**
6.	*(a)*	$2x^2 + 3\ln(2 - x) = 0 \;\Rightarrow\; 3\ln(2 - x) = -2x^2$		
		$\ln(2 - x) = -\frac{2}{3}x^2$	M1	
		$2 - x = e^{-\frac{2}{3}x^2}$	M1	
		$x = 2 - e^{-\frac{2}{3}x^2} \quad [\, k = -\frac{2}{3} \,]$	A1	
	(b)	$x_1 = 1.90988, x_2 = 1.91212, x_3 = 1.91262, x_4 = 1.91273$	M1 A1	
		$\therefore \; \alpha = 1.913$ (3dp)	A1	
		$f(1.9125) = 0.0070, \; f(1.9135) = -0.020$	M1	
		sign change, $f(x)$ continuous $\therefore$ root	A1	
	(c)	$f'(x) = 4x + \frac{3}{2-x} \times (-1) = 4x - \frac{3}{2-x}$	M1 A1	
		$\therefore \; 4x - \frac{3}{2-x} = 0, \quad 4x = \frac{3}{2-x}, \quad 4x(2 - x) = 3$	M1	
		$4x^2 - 8x + 3 = 0, \quad (2x - 3)(2x - 1) = 0$	M1	
		$x = \frac{1}{2}, \frac{3}{2}$	A1	**(13)**
7.	*(a)*	*(i)* 	B3	
		(ii) 	M1 A2	
	(b)	$2\sqrt{2}\cos x - 2\sqrt{2}\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
		$R\cos\alpha = 2\sqrt{2}, \; R\sin\alpha = 2\sqrt{2}, \quad \therefore \; R = \sqrt{8+8} = 4$	M1 A1	
		$\tan\alpha = 1, \; \alpha = 45$	B1	
		$\therefore \; f(x) = A + 4\cos(x + 45)°$		
	(c)	3	B1	
	(d)	$3 + 4\cos(x + 45) = 0, \quad \cos(x + 45) = -\frac{3}{4}$	M1	
		$x + 45 = 180 - 41.4, \; 180 + 41.4 = 138.6, \; 221.4$	M1	
		$x = 93.6, \; 176.4$ (1dp)	A2	**(14)**
			Total	**(75)**