1. *(a)* Given that $\cos x = \sqrt{3} - 1$, find the value of $\cos 2x$ in the form $a + b\sqrt{3}$, where a and b are integers. **(3)**

(b) Given that

$$2 \cos (y + 30)° = \sqrt{3} \sin (y - 30)°,$$

find the value of $\tan y$ in the form $k\sqrt{3}$ where k is a rational constant. **(5)**

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2. The functions f and g are defined by

$$\mathrm{f}(x) \equiv x^2 - 3x + 7, \quad x \in \mathbb{R},$$

$$\mathrm{g}(x) \equiv 2x - 1, \quad x \in \mathbb{R}.$$

(a) Find the range of f. **(3)**

(b) Evaluate gf(−1). **(2)**

(c) Solve the equation

$$\mathrm{fg}(x) = 17.$$ **(4)**

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3. $$\mathrm{f}(x) = \frac{x^4 + x^3 - 13x^2 + 26x - 17}{x^2 - 3x + 3}, \quad x \in \mathbb{R}.$$

(a) Find the values of the constants A, B, C and D such that

$$\mathrm{f}(x) = x^2 + Ax + B + \frac{Cx + D}{x^2 - 3x + 3}.$$ **(4)**

The point P on the curve $y = \mathrm{f}(x)$ has x-coordinate 1.

(b) Show that the normal to the curve $y = \mathrm{f}(x)$ at P has the equation

$$x + 5y + 9 = 0.$$ **(6)**

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4. *(a)* Given that

$$x = \sec \frac{y}{2}, \quad 0 \leq y < \pi,$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{x\sqrt{x^2 - 1}}. \qquad \textbf{(5)}$$

(b) Find an equation for the tangent to the curve $y = \sqrt{3 + 2\cos x}$ at the point where $x = \frac{\pi}{3}$. **(6)**

---

5. $$\mathrm{f}(x) = 5 + \mathrm{e}^{2x-3}, \quad x \in \mathbb{R}.$$

(a) State the range of f. **(1)**

(b) Find an expression for $\mathrm{f}^{-1}(x)$ and state its domain. **(4)**

(c) Solve the equation $\mathrm{f}(x) = 7$. **(2)**

(d) Find an equation for the tangent to the curve $y = \mathrm{f}(x)$ at the point where $y = 7$. **(4)**

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6. *(a)* Prove the identity

$$2\cot 2x + \tan x \equiv \cot x, \quad x \neq \frac{n}{2}\pi, \quad n \in \mathbb{Z}. \qquad \textbf{(5)}$$

(b) Solve, for $0 \leq x < \pi$, the equation

$$2\cot 2x + \tan x = \operatorname{cosec}^2 x - 7,$$

giving your answers to 2 decimal places. **(6)**

Turn over

7. The functions f and g are defined by

$$\mathrm{f} : x \to |2x - 5|, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \to \ln (x + 3), \quad x \in \mathbb{R}, \quad x > -3.$$

(a) State the range of f. **(1)**

(b) Evaluate fg(–2). **(2)**

(c) Solve the equation

$$\mathrm{fg}(x) = 3,$$

giving your answers in exact form. **(5)**

(d) Show that the equation

$$\mathrm{f}(x) = \mathrm{g}(x)$$

has a root, α, in the interval [3, 4]. **(2)**

(e) Use the iteration formula

$$x_{n+1} = \tfrac{1}{2}[5 + \ln (x_n + 3)],$$

with $x_0 = 3$, to find x_1, x_2, x_3 and x_4, giving your answers to 4 significant figures. **(3)**

(f) Show that your answer for x_4 is the value of α correct to 4 significant figures. **(2)**

END

C3 Paper J – Marking Guide

1. *(a)* $\cos^2 x = (\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$ — M1

$\cos 2x = 2\cos^2 x - 1 = 2(4 - 2\sqrt{3}) - 1 = 7 - 4\sqrt{3}$ — M1 A1

(b) $2(\cos y \cos 30 - \sin y \sin 30) = \sqrt{3}(\sin y \cos 30 - \cos y \sin 30)$ — M1 A1

$\sqrt{3}\cos y - \sin y = \frac{3}{2}\sin y - \frac{1}{2}\sqrt{3}\cos y$ — B1

$\frac{3}{2}\sqrt{3}\cos y = \frac{5}{2}\sin y$

$\tan y = \frac{3}{2}\sqrt{3} \div \frac{5}{2} = \frac{3}{5}\sqrt{3}$ — M1 A1 **(8)**

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2. *(a)* $\text{f}(x) = (x - \frac{3}{2})^2 - \frac{9}{4} + 7 = (x - \frac{3}{2})^2 + \frac{19}{4}$ — M1 A1

$\therefore\ \text{f}(x) \geq \frac{19}{4}$ — A1

(b) $= \text{g}(11) = 21$ — M1 A1

(c) $\text{fg}(x) = \text{f}(2x - 1) = (2x - 1)^2 - 3(2x - 1) + 7$ — M1

$\therefore\ 4x^2 - 4x + 1 - 6x + 3 + 7 = 17$

$2x^2 - 5x - 3 = 0$ — A1

$(2x + 1)(x - 3) = 0$ — M1

$x = -\frac{1}{2}, 3$ — A1 **(9)**

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3. *(a)*

$$\begin{array}{r|rrrrr}
 & x^2 & +\,4x & -\,4 & & \\ \hline
x^2 - 3x + 3 & x^4 + x^3 & -\,13x^2 & +\,26x & -\,17 & \\
 & x^4 - 3x^3 & +\,3x^2 & & & \\ \hline
 & 4x^3 & -\,16x^2 & +\,26x & & \\
 & 4x^3 & -\,12x^2 & +\,12x & & \\ \hline
 & & -\,4x^2 & +\,14x & -\,17 & \\
 & & -\,4x^2 & +\,12x & -\,12 & \\ \hline
 & & & 2x & -\,5 &
\end{array}$$

— M1

$\therefore\ \text{f}(x) = x^2 + 4x - 4 + \frac{2x-5}{x^2-3x+3}$, $A = 4, B = -4, C = 2, D = -5$ — A3

(b) $\text{f}'(x) = 2x + 4 + \frac{2\times(x^2-3x+3)-(2x-5)\times(2x-3)}{(x^2-3x+3)^2}$ — M1 A2

$x = 1 \Rightarrow y = -2$, grad $= 5$

$\therefore$ grad of normal $= -\frac{1}{5}$ — M1

$\therefore\ y + 2 = -\frac{1}{5}(x - 1)$ — M1

$5y + 10 = -x + 1$

$x + 5y + 9 = 0$ — A1 **(10)**

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4. *(a)* $\frac{\text{d}x}{\text{d}y} = \frac{1}{2}\sec\frac{y}{2}\tan\frac{y}{2}$ — M1

$0 \leq y < \pi\ \therefore\ \tan\frac{y}{2} \geq 0\ \therefore\ \frac{\text{d}x}{\text{d}y} = \frac{1}{2}\sec\frac{y}{2}\sqrt{\sec^2\frac{y}{2} - 1} = \frac{1}{2}x\sqrt{x^2 - 1}$ — M1 A1

$\frac{\text{d}y}{\text{d}x} = 1 \div \frac{\text{d}x}{\text{d}y} = \frac{2}{x\sqrt{x^2-1}}$ — M1 A1

(b) $\frac{\text{d}y}{\text{d}x} = \frac{1}{2}(3 + 2\cos x)^{-\frac{1}{2}} \times (-2\sin x) = -\frac{\sin x}{\sqrt{3+2\cos x}}$ — M1 A1

$x = \frac{\pi}{3}$, $y = 2$, grad $= -\frac{1}{4}\sqrt{3}$ — M1 A1

$\therefore\ y - 2 = -\frac{1}{4}\sqrt{3}(x - \frac{\pi}{3})$ $[\ 3\sqrt{3}x + 12y - 24 - \pi\sqrt{3} = 0\ ]$ — M1 A1 **(11)**

5.	*(a)*	$f(x) > 5$	B1	
	(b)	$y = 5 + e^{2x-3}$		
		$2x - 3 = \ln(y - 5)$	M1	
		$x = \frac{1}{2}[3 + \ln(y - 5)]$	M1	
		$\therefore\ f^{-1}(x) = \frac{1}{2}[3 + \ln(x - 5)],\ x \in \mathbb{R},\ x > 5$	A2	
	(c)	$x = f^{-1}(7) = \frac{1}{2}(3 + \ln 2)$	M1 A1	
	(d)	$f'(x) = 2e^{2x-3}$	M1	
		grad = 4	A1	
		$\therefore\ y - 7 = 4[x - \frac{1}{2}(3 + \ln 2)]$ $\quad [\, y = 4x + 1 - 2\ln 2 \,]$	M1 A1	**(11)**

6.	*(a)*	LHS $\equiv \dfrac{2\cos 2x}{\sin 2x} + \dfrac{\sin x}{\cos x}$	M1	
		$\equiv \dfrac{\cos 2x}{\sin x \cos x} + \dfrac{\sin x}{\cos x}$	M1	
		$\equiv \dfrac{\cos 2x + \sin^2 x}{\sin x \cos x}$	A1	
		$\equiv \dfrac{(\cos^2 x - \sin^2 x) + \sin^2 x}{\sin x \cos x}$	M1	
		$\equiv \dfrac{\cos^2 x}{\sin x \cos x}$		
		$\equiv \dfrac{\cos x}{\sin x}$		
		$\equiv \cot x \equiv$ RHS	A1	
	(b)	$\cot x = \operatorname{cosec}^2 x - 7$		
		$\cot x = 1 + \cot^2 x - 7$	M1	
		$\cot^2 x - \cot x - 6 = 0$		
		$(\cot x + 2)(\cot x - 3) = 0$	M1	
		$\cot x = -2$ or 3	A1	
		$\tan x = -\frac{1}{2}$ or $\frac{1}{3}$	M1	
		$x = \pi - 0.4636$ or 0.32		
		$x = 0.32, 2.68$ (2dp)	A2	**(11)**

7.	*(a)*	$f(x) \geq 0$	B1	
	(b)	$= f(0) = 5$	M1 A1	
	(c)	$fg(x) = f[\ln(x + 3)] = \lvert 2\ln(x + 3) - 5 \rvert$	M1	
		$\therefore\ \lvert 2\ln(x + 3) - 5 \rvert = 3$		
		$2\ln(x + 3) = 2, 8$	M1	
		$\ln(x + 3) = 1, 4$	A1	
		$x = e - 3, e^4 - 3$	M1 A1	
	(d)	let $h(x) = f(x) - g(x)$		
		$h(3) = -0.79,\ f(4) = 1.1$	M1	
		sign change, $h(x)$ continuous $\therefore$ root	A1	
	(e)	$x_1 = 3.396, x_2 = 3.428, x_3 = 3.430, x_4 = 3.431$	M1 A2	
	(f)	$h(3.4305) = -0.000052,\ f(3.4315) = 0.0018$	M1	
		sign change, $h(x)$ continuous $\therefore$ root $\therefore\ \alpha = x_4$ to 4sf	A1	**(15)**

Total **(75)**