1. Express

$$\frac{2x}{2x^2+3x-5} \div \frac{x^3}{x^2-x}$$

as a single fraction in its simplest form. **(5)**

2.

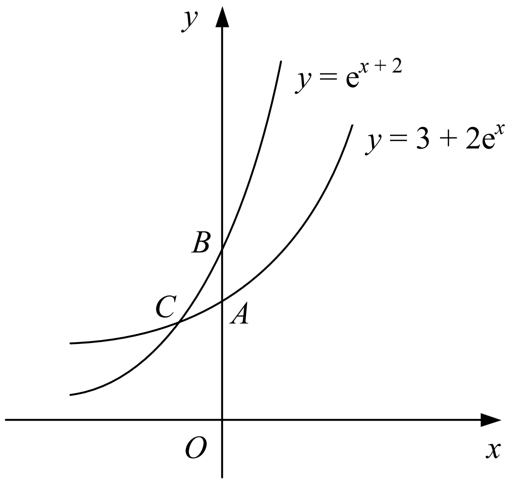

Figure 1

Figure 1 shows the curves $y = 3 + 2e^x$ and $y = e^{x+2}$ which cross the y-axis at the points A and B respectively.

(a) Find the exact length AB. **(3)**

The two curves intersect at the point C.

(b) Find an expression for the x-coordinate of C and show that the y-coordinate of C is $\frac{3e^2}{e^2-2}$. **(5)**

3.

$$f(x) = \frac{x^2+3}{4x+1}, \quad x \in \mathbb{R}, \quad x \neq -\tfrac{1}{4}.$$

(a) Find and simplify an expression for $f'(x)$. **(3)**

(b) Find the set of values of x for which $f(x)$ is increasing. **(5)**

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4. The curve C has the equation $y = x^2 - 5x + 2 \ln \frac{x}{3}$, $x > 0$.

(a) Show that the normal to C at the point where $x = 3$ has the equation

$$3x + 5y + 21 = 0.$$ **(5)**

(b) Find the x-coordinates of the stationary points of C. **(3)**

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5. The functions f and g are defined by

$$\mathrm{f}(x) \equiv 6x - 1, \quad x \in \mathbb{R},$$

$$\mathrm{g}(x) \equiv \log_2 (3x + 1), \quad x \in \mathbb{R}, \quad x > -\tfrac{1}{3}.$$

(a) Evaluate gf(1). **(2)**

(b) Find an expression for $\mathrm{g}^{-1}(x)$. **(3)**

(c) Find, in terms of natural logarithms, the solution of the equation

$$\mathrm{fg}^{-1}(x) = 2.$$ **(4)**

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6. *(a)* Use the identities for $\cos (A + B)$ and $\cos (A - B)$ to prove that

$$\cos P - \cos Q \equiv -2 \sin \frac{P+Q}{2} \sin \frac{P-Q}{2}.$$ **(4)**

(b) Hence find all solutions in the interval $0 \le x < 180$ to the equation

$$\cos 5x° + \sin 3x° - \cos x° = 0.$$ **(7)**

Turn over

7. The function f is defined by

$$f(x) \equiv x^2 - 2ax, \quad x \in \mathbb{R},$$

where a is a positive constant.

(a) Showing the coordinates of any points where each graph meets the axes, sketch on separate diagrams the graphs of

(i) $y = |f(x)|$,

(ii) $y = f(|x|)$. **(6)**

The function g is defined by

$$g(x) \equiv 3ax, \quad x \in \mathbb{R}.$$

(b) Find $fg(a)$ in terms of a. **(2)**

(c) Solve the equation

$$gf(x) = 9a^3.$$ **(4)**

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8. $$f(x) = 2x + \sin x - 3 \cos x.$$

(a) Show that the equation $f(x) = 0$ has a root in the interval $[0.7, 0.8]$. **(2)**

(b) Find an equation for the tangent to the curve $y = f(x)$ at the point where it crosses the y-axis. **(4)**

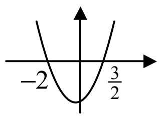
(c) Find the values of the constants a, b and c, where $b > 0$ and $0 < c < \frac{\pi}{2}$, such that

$$f'(x) = a + b \cos (x - c).$$ **(4)**

(d) Hence find the x-coordinates of the stationary points of the curve $y = f(x)$ in the interval $0 \leq x \leq 2\pi$, giving your answers to 2 decimal places. **(4)**

END

C3 Paper I – Marking Guide

1.		$= \dfrac{2x}{2x^2+3x-5} \times \dfrac{x^2-x}{x^3}$	M1	
		$= \dfrac{2x}{(2x+5)(x-1)} \times \dfrac{x(x-1)}{x^3}$	M1 A1	
		$= \dfrac{2}{x(2x+5)}$	M1 A1	**(5)**
2.	*(a)*	$A\ (0,\ 5),\ \ B\ (0,\ \mathrm{e}^2)$	B2	
		$\therefore\ AB = \mathrm{e}^2 - 5$	B1	
	(b)	$3 + 2\mathrm{e}^x = \mathrm{e}^{x+2} = \mathrm{e}^2\mathrm{e}^x$	M1	
		$3 = \mathrm{e}^x(\mathrm{e}^2 - 2)$		
		$\mathrm{e}^x = \dfrac{3}{\mathrm{e}^2-2}, \qquad x = \ln \dfrac{3}{\mathrm{e}^2-2}$	M1 A1	
		$\therefore\ y = \mathrm{e}^2\mathrm{e}^x = \mathrm{e}^2 \times \dfrac{3}{\mathrm{e}^2-2} = \dfrac{3\mathrm{e}^2}{\mathrm{e}^2-2}$	M1 A1	**(8)**
3.	*(a)*	$\mathrm{f}'(x) = \dfrac{2x\times(4x+1)-(x^2+3)\times 4}{(4x+1)^2} = \dfrac{4x^2+2x-12}{(4x+1)^2}$	M1 A2	
	(b)	$\dfrac{4x^2+2x-12}{(4x+1)^2} \geq 0$	M1	
		for $x \neq -\frac{1}{4}$, $(4x+1)^2 > 0\ \ \therefore\ 4x^2 + 2x - 12 \geq 0$ 	M1 A1	
		$2(2x-3)(x+2) \geq 0$	M1	
		$x \leq -2$ or $x \geq \frac{3}{2}$	A1	**(8)**
4.	*(a)*	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x - 5 + \dfrac{2}{x}$	M1	
		$x = 3,\ y = -6,$ grad $= \frac{5}{3}$	A1	
		grad of normal $= -\frac{3}{5}$	M1	
		$\therefore\ y + 6 = -\frac{3}{5}(x - 3)$	M1	
		$5y + 30 = -3x + 9$		
		$3x + 5y + 21 = 0$	A1	
	(b)	SP: $\quad 2x - 5 + \dfrac{2}{x} = 0$		
		$2x^2 - 5x + 2 = 0$	M1	
		$(2x-1)(x-2) = 0$	M1	
		$x = \frac{1}{2},\ 2$	A1	**(8)**
5.	*(a)*	$= \mathrm{g}(5) = \log_2 16 = 4$	M1 A1	
	(b)	$y = \log_2 (3x + 1)$		
		$3x + 1 = 2^y$	M1	
		$x = \frac{1}{3}(2^y - 1)$		
		$\mathrm{g}^{-1}(x) = \frac{1}{3}(2^x - 1)$	M1 A1	
	(c)	$\mathrm{fg}^{-1}(x) = \mathrm{f}\,[\frac{1}{3}(2^x - 1)] = 2(2^x - 1) - 1 = 2(2^x) - 3$	M1	
		$\therefore\ 2(2^x) - 3 = 2$		
		$2^x = \frac{5}{2}$	A1	
		$x = \dfrac{\ln \frac{5}{2}}{\ln 2}$ or $\dfrac{\ln 5 - \ln 2}{\ln 2}$	M1 A1	**(9)**

6. *(a)* $\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$

$\cos(A-B) \equiv \cos A \cos B + \sin A \sin B$

subtracting, $\cos(A+B) - \cos(A-B) \equiv -2 \sin A \sin B$ M1 A1

let $P = A + B,\ Q = A - B$

adding, $P + Q = 2A \Rightarrow A = \frac{P+Q}{2}$ M1

subtracting, $P - Q = 2B \Rightarrow B = \frac{P-Q}{2}$

$\therefore\ \cos P - \cos Q \equiv -2 \sin \frac{P+Q}{2} \sin \frac{P-Q}{2}$ A1

(b) $(\cos 5x - \cos x) + \sin 3x = 0$

$-2 \sin 3x \sin 2x + \sin 3x = 0$ M1

$\sin 3x(1 - 2 \sin 2x) = 0$ M1

$\sin 3x = 0$ or $\sin 2x = \frac{1}{2}$ A1

$3x = 0, 180, 360$ or $2x = 30, 150$ B1

$x = 0, 15, 60, 75, 120$ M1 A2 **(11)**

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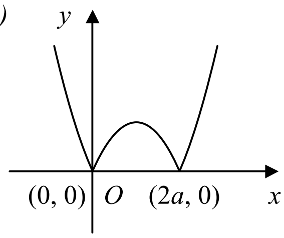
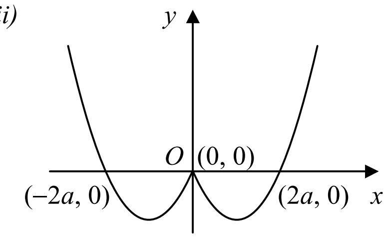
7. *(a)* *(i)*



B3

(ii)



B3

(b) $= f(3a^2) = 9a^4 - 6a^3$ M1 A1

(c) $gf(x) = 3a(x^2 - 2ax)$ M1

$\therefore\ 3a(x^2 - 2ax) = 9a^3$

$x^2 - 2ax - 3a^2 = 0$ A1

$(x + a)(x - 3a) = 0$ M1

$x = -a, 3a$ A1 **(12)**

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8. *(a)* $f(0.7) = -0.25,\ f(0.8) = 0.23$ M1

sign change, $f(x)$ continuous $\therefore$ root A1

(b) $f'(x) = 2 + \cos x + 3 \sin x$ M1

$x = 0,\ y = -3,$ grad $= 3$ A1

$\therefore\ y = 3x - 3$ M1 A1

(c) $\cos x + 3 \sin x = b \cos x \cos c + b \sin x \sin c$

$b \cos c = 1,\ b \sin c = 3$

$\therefore\ b = \sqrt{1^2 + 3^2} = \sqrt{10}$ M1

$\tan c = 3,\ c = 1.25$ (3sf) M1

$\therefore\ a = 2, b = \sqrt{10}, c = 1.25$ A2

(d) SP: $2 + \sqrt{10} \cos(x - 1.249) = 0$

$\cos(x - 1.249) = -\frac{2}{\sqrt{10}}$ M1

$x - 1.249 = \pi - 0.8861, \pi + 0.8861 = 2.256, 4.028$ M1

$x = 3.50, 5.28$ (2dp) A2 **(14)**

Total **(75)**