1. The functions f and g are defined by

$$f : x \to 2 - x^2, \quad x \in \mathbb{R},$$

$$g : x \to \frac{3x}{2x-1}, \quad x \in \mathbb{R}, \quad x \neq \tfrac{1}{2}.$$

(a) Evaluate fg(2). **(2)**

(b) Solve the equation $gf(x) = \frac{1}{2}$. **(4)**

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2. Giving your answers to 1 decimal place, solve the equation

$$5 \tan^2 2\theta - 13 \sec 2\theta = 1,$$

for $\theta$ in the interval $0 \leq \theta \leq 360°$. **(7)**

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3. (a) Simplify

$$\frac{2x^2 + 3x - 9}{2x^2 - 7x + 6}.$$ **(3)**

(b) Solve the equation

$$\ln (2x^2 + 3x - 9) = 2 + \ln (2x^2 - 7x + 6),$$

giving your answer in terms of e. **(4)**

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**4.**

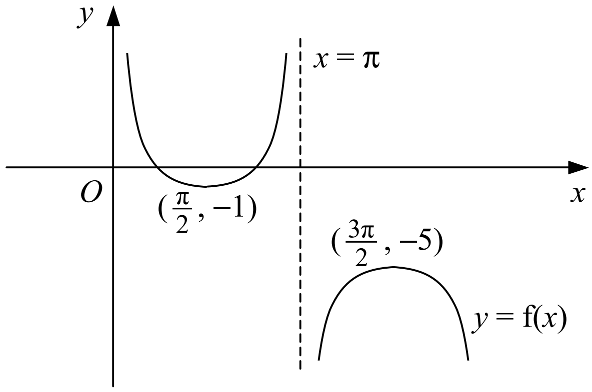


**Figure 1**

Figure 1 shows the graph of $y = \mathrm{f}(x)$. The graph has a minimum at $(\frac{\pi}{2}, -1)$, a maximum at $(\frac{3\pi}{2}, -5)$ and an asymptote with equation $x = \pi$.

*(a)* Showing the coordinates of any stationary points, sketch the graph of $y = |\mathrm{f}(x)|$. **(3)**

Given that

$$\mathrm{f} : x \rightarrow a + b\operatorname{cosec} x, \quad x \in \mathbb{R}, \quad 0 < x < 2\pi, \quad x \neq \pi,$$

*(b)* find the values of the constants $a$ and $b$, **(3)**

*(c)* find, to 2 decimal places, the $x$-coordinates of the points where the graph of $y = \mathrm{f}(x)$ crosses the $x$-axis. **(3)**

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**5.** The number of bacteria present in a culture at time $t$ hours is modelled by the continuous variable $N$ and the relationship

$$N = 2000\mathrm{e}^{kt},$$

where $k$ is a constant.

Given that when $t = 3$, $N = 18\,000$, find

*(a)* the value of $k$ to 3 significant figures, **(3)**

*(b)* how long it takes for the number of bacteria present to double, giving your answer to the nearest minute, **(4)**

*(c)* the rate at which the number of bacteria is increasing when $t = 3$. **(3)**

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***Turn over***

6. *(a)* Use the derivative of $\cos x$ to prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \sec x \tan x.$$ **(4)**

The curve $C$ has the equation $y = \mathrm{e}^{2x} \sec x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$.

*(b)* Find an equation for the tangent to $C$ at the point where it crosses the $y$-axis. **(4)**

*(c)* Find, to 2 decimal places, the $x$-coordinate of the stationary point of $C$. **(3)**

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7. $$\mathrm{f}(x) = x^2 - 2x + 5,\quad x \in \mathbb{R},\quad x \geq 1.$$

*(a)* Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b$, where $a$ and $b$ are constants. **(2)**

*(b)* State the range of f. **(1)**

*(c)* Find an expression for $\mathrm{f}^{-1}(x)$. **(3)**

*(d)* Describe fully two transformations that would map the graph of $y = \mathrm{f}^{-1}(x)$ onto the graph of $y = \sqrt{x}$, $x \geq 0$. **(2)**

*(e)* Find an equation for the normal to the curve $y = \mathrm{f}^{-1}(x)$ at the point where $x = 8$. **(4)**

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8. A curve has the equation $y = \frac{\mathrm{e}^2}{x} + \mathrm{e}^x,\quad x \neq 0$.

*(a)* Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **(2)**

*(b)* Show that the curve has a stationary point in the interval $[1.3, 1.4]$. **(3)**

The point $A$ on the curve has $x$-coordinate 2.

*(c)* Show that the tangent to the curve at $A$ passes through the origin. **(4)**

The tangent to the curve at $A$ intersects the curve again at the point $B$.

The $x$-coordinate of $B$ is to be estimated using the iterative formula

$$x_{n+1} = -\tfrac{2}{3}\sqrt{3 + 3x_n \mathrm{e}^{x_n - 2}}\,,$$

with $x_0 = -1$.

*(d)* Find $x_1$, $x_2$ and $x_3$ to 7 significant figures and hence state the $x$-coordinate of $B$ to 5 significant figures. **(4)**

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**END**

# C3 Paper H – Marking Guide

**1.** *(a)* $= f(2) = -2$ M1 A1

*(b)* $gf(x) = g(2 - x^2) = \frac{3(2-x^2)}{2(2-x^2)-1} = \frac{6-3x^2}{3-2x^2}$ M1 A1

$\therefore\ \frac{6-3x^2}{3-2x^2} = \frac{1}{2}, \quad 2(6 - 3x^2) = 3 - 2x^2$

$x^2 = \frac{9}{4}, \quad x = \pm\frac{3}{2}$ M1 A1 **(6)**

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**2.** $5(\sec^2 2\theta - 1) - 13\sec 2\theta = 1$ M1

$5\sec^2 2\theta - 13\sec 2\theta - 6 = 0$

$(5\sec 2\theta + 2)(\sec 2\theta - 3) = 0$ M1

$\sec 2\theta = -\frac{2}{5}$ or $3$ A1

$\cos 2\theta = -\frac{5}{2}$ (no solutions) or $\frac{1}{3}$

$2\theta = 70.529,\ 360 - 70.529,\ 360 + 70.529,\ 720 - 70.529$ B1 M1

$= 70.529,\ 289.471,\ 430.529,\ 649.471$

$\theta = 35.3°,\ 144.7°,\ 215.3°,\ 324.7°$ (1dp) A2 **(7)**

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**3.** *(a)* $= \frac{(2x-3)(x+3)}{(2x-3)(x-2)} = \frac{x+3}{x-2}$ M1 A2

*(b)* $\ln(2x^2 + 3x - 9) - \ln(2x^2 - 7x + 6) = 2, \quad \ln\frac{2x^2+3x-9}{2x^2-7x+6} = 2$ M1

$\ln\frac{x+3}{x-2} = 2, \quad \frac{x+3}{x-2} = e^2$ A1

$x + 3 = e^2(x - 2)$

$3 + 2e^2 = x(e^2 - 1)$ M1

$x = \frac{2e^2+3}{e^2-1}$ A1 **(7)**

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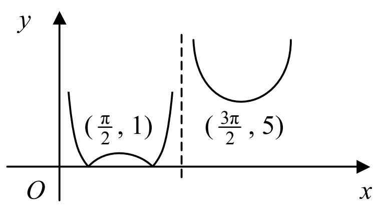
**4.** *(a)*




B3

*(b)* $(\frac{\pi}{2}, -1) \Rightarrow -1 = a + b$

$(\frac{3\pi}{2}, -5) \Rightarrow -5 = a - b$ B1

adding, $-6 = 2a \ \therefore\ a = -3,\ b = 2$ M1 A1

*(c)* $-3 + 2\operatorname{cosec} x = 0, \quad \operatorname{cosec} x = \frac{3}{2}, \quad \sin x = \frac{2}{3}$ M1

$x = 0.73,\ \pi - 0.7297, \quad x = 0.73,\ 2.41$ (2dp) A2 **(9)**

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**5.** *(a)* $t = 3,\ N = 18\,000 \Rightarrow 18\,000 = 2000e^{3k}, \quad e^{3k} = 9$ M1

$k = \frac{1}{3}\ln 9 = 0.732$ (3sf) M1 A1

*(b)* $4000 = 2000e^{0.7324t}$ M1

$t = \frac{1}{0.7324}\ln 2 = 0.9464$ hours M1 A1

$\therefore$ doubles in 57 minutes (nearest minute) A1

*(c)* $N = 2000e^{0.7324t}, \quad \frac{dN}{dt} = 0.7324 \times 2000e^{0.7324t} = 1465e^{0.7324t}$ M1 A1

when $t = 3$, $\frac{dN}{dt} = 13\,200$ $\therefore$ increasing at rate of 13 200 per hour (3sf) A1 **(10)**

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**6.** *(a)* $\frac{d}{dx}(\sec x) = \frac{d}{dx}[(\cos x)^{-1}]$

$= -(\cos x)^{-2} \times (-\sin x)$ — M1 A1

$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$ — M1

$= \sec x \tan x$ — A1

*(b)* $\frac{dy}{dx} = 2e^{2x} \times \sec x + e^{2x} \times \sec x \tan x = e^{2x} \sec x\,(2 + \tan x)$ — M1 A1

$x = 0,\ y = 1,\ \text{grad} = 2$ — M1

$\therefore\ y = 2x + 1$ — A1

*(c)* SP: $e^{2x} \sec x\,(2 + \tan x) = 0$ — M1

$\tan x = -2$ — M1

$x = -1.11$ (2dp) — A1 **(11)**

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**7.** *(a)* $f(x) = (x-1)^2 - 1 + 5 = (x-1)^2 + 4$ — M1 A1

*(b)* $f(x) \geq 4$ — B1

*(c)* $y = (x-1)^2 + 4$

$(x-1)^2 = y - 4$

$x - 1 = \pm\sqrt{y-4}$ — M1

$x = 1 \pm \sqrt{y-4}$

$f^{-1}(x) = 1 + \sqrt{x-4}$ — M1 A1

*(d)* translation by 4 units in negative $x$ direction

translation by 1 unit in negative $y$ direction (either first) — B2

*(e)* $\frac{dy}{dx} = \frac{1}{2}(x-4)^{-\frac{1}{2}}$ — M1

$x = 8,\ y = 3,\ \text{grad} = \frac{1}{4}$ — A1

$\therefore$ grad of normal $= -4$

$\therefore\ y - 3 = -4(x - 8)$ $\quad [\, y = 35 - 4x \,]$ — M1 A1 **(12)**

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**8.** *(a)* $\frac{dy}{dx} = -e^2x^{-2} + e^x$ — M1 A1

*(b)* SP: $-e^2x^{-2} + e^x = 0$ — M1

let $f(x) = -e^2x^{-2} + e^x$

$f(1.3) = -0.70,\ f(1.4) = 0.29$ — M1

sign change, $f(x)$ continuous $\therefore$ root — A1

*(c)* $x = 2,\ y = \frac{3}{2}e^2,\ \text{grad} = \frac{3}{4}e^2$ — M1

$\therefore\ y - \frac{3}{2}e^2 = \frac{3}{4}e^2(x - 2)$ — M1 A1

$y = \frac{3}{4}e^2 x$

$\therefore\ x = 0 \Rightarrow y = 0$ so passes through origin — A1

*(d)* $x_1 = -1.125589,\ x_2 = -1.125803,\ x_3 = -1.125804$ (7sf) — M1 A2

$\therefore$ $x$-coordinate of $B = -1.1258$ (5sf) — A1 **(13)**

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Total **(75)**