1. A curve has the equation $y = (3x - 5)^3$.

(a) Find an equation for the tangent to the curve at the point P (2, 1). **(4)**

The tangent to the curve at the point Q is parallel to the tangent at P.

(b) Find the coordinates of Q. **(3)**

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2. *(a)* Use the identities for $\cos(A + B)$ and $\cos(A - B)$ to prove that

$$2\cos A\cos B \equiv \cos(A + B) + \cos(A - B).$$ **(2)**

(b) Hence, or otherwise, find in terms of π the solutions of the equation

$$2\cos\left(x + \tfrac{\pi}{2}\right) = \sec\left(x + \tfrac{\pi}{6}\right),$$

for x in the interval $0 \leq x \leq \pi$. **(7)**

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3. Differentiate each of the following with respect to x and simplify your answers.

(a) $\ln(\cos x)$ **(3)**

(b) $x^2 \sin 3x$ **(3)**

(c) $\dfrac{6}{\sqrt{2x-7}}$ **(4)**

---

4. *(a)* Express $2\sin x° - 3\cos x°$ in the form $R\sin(x - \alpha)°$ where $R > 0$ and $0 < \alpha < 90$. **(4)**

(b) Show that the equation

$$\operatorname{cosec} x° + 3\cot x° = 2$$

can be written in the form

$$2\sin x° - 3\cos x° = 1.$$ **(1)**

(c) Solve the equation

$$\operatorname{cosec} x° + 3\cot x° = 2,$$

for x in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place. **(5)**

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5. *(a)* Show that $(2x + 3)$ is a factor of $(2x^3 - x^2 + 4x + 15)$. **(2)**

(b) Hence, simplify

$$\frac{2x^2 + x - 3}{2x^3 - x^2 + 4x + 15}.$$ **(4)**

(c) Find the coordinates of the stationary points of the curve with equation

$$y = \frac{2x^2 + x - 3}{2x^3 - x^2 + 4x + 15}.$$ **(6)**

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6. The population in thousands, P, of a town at time t years after 1st January 1980 is modelled by the formula

$$P = 30 + 50e^{0.002t}.$$

Use this model to estimate

(a) the population of the town on 1st January 2010, **(2)**

(b) the year in which the population first exceeds 84 000. **(4)**

The population in thousands, Q, of another town is modelled by the formula

$$Q = 26 + 50e^{0.003t}.$$

(c) Show that the value of t when $P = Q$ is a solution of the equation

$$t = 1000 \ln (1 + 0.08e^{-0.002t}).$$ **(3)**

(d) Use the iteration formula

$$t_{n+1} = 1000 \ln (1 + 0.08e^{-0.002t_n})$$

with $t_0 = 50$ to find t_1, t_2 and t_3 and hence, the year in which the populations of these two towns will be equal according to these models. **(4)**

Turn over

7.

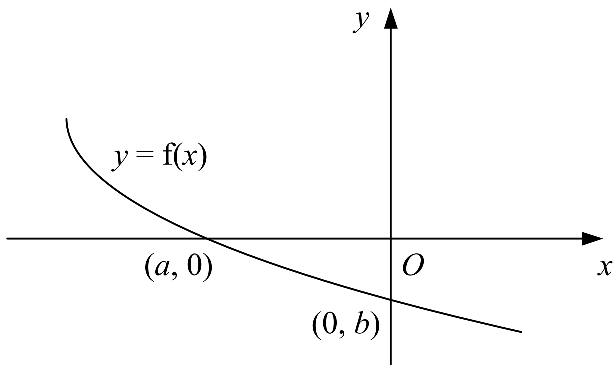


Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$ which meets the coordinate axes at the points $(a, 0)$ and $(0, b)$, where a and b are constants.

(a) Showing, in terms of a and b, the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

(i) $y = \mathrm{f}^{-1}(x)$,

(ii) $y = 2\mathrm{f}(3x)$. **(6)**

Given that

$$\mathrm{f}(x) = 2 - \sqrt{x+9}, \quad x \in \mathbb{R}, \quad x \geq -9,$$

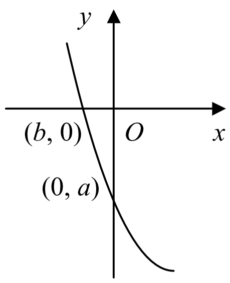
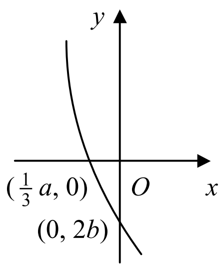
(b) find the values of a and b, **(3)**

(c) find an expression for $\mathrm{f}^{-1}(x)$ and state its domain. **(5)**

END

C3 Paper G – Marking Guide

1.	*(a)*	$\frac{dy}{dx} = 3(3x-5)^2 \times 3 = 9(3x-5)^2$	M1	
		grad = 9	A1	
		$\therefore\ y - 1 = 9(x-2)$ $\quad [\ y = 9x - 17\ ]$	M1 A1	
	(b)	$9(3x-5)^2 = 9$		
		$3x - 5 = \pm 1$	M1	
		$x = 2$ (at P), $\frac{4}{3}$	A1	
		$\therefore\ Q\,(\frac{4}{3}, -1)$	A1	**(7)**
2.	*(a)*	$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$		
		$\cos(A-B) \equiv \cos A \cos B + \sin A \sin B$		
		adding, $\quad 2\cos A \cos B \equiv \cos(A+B) + \cos(A-B)$	M1 A1	
	(b)	$2\cos(x + \frac{\pi}{2})\cos(x + \frac{\pi}{6}) = 1$	M1	
		$\cos(2x + \frac{2\pi}{3}) + \cos\frac{\pi}{3} = 1$	M1	
		$\cos(2x + \frac{2\pi}{3}) = 1 - \frac{1}{2} = \frac{1}{2}$	A1	
		$2x + \frac{2\pi}{3} = 2\pi - \frac{\pi}{3}, 2\pi + \frac{\pi}{3} = \frac{5\pi}{3}, \frac{7\pi}{3}$	B1	
		$2x = \pi, \frac{5\pi}{3}$	M1	
		$x = \frac{\pi}{2}, \frac{5\pi}{6}$	A2	**(9)**
3.	*(a)*	$= \frac{1}{\cos x} \times (-\sin x) = -\tan x$	M1 A2	
	(b)	$= 2x \times \sin 3x + x^2 \times 3\cos 3x = 2x\sin 3x + 3x^2\cos 3x$	M1 A2	
	(c)	$= \frac{d}{dx}[\,6(2x-7)^{-\frac{1}{2}}\,]$	B1	
		$= -3(2x-7)^{-\frac{3}{2}} \times 2 = -\frac{6}{(2x-7)^{\frac{3}{2}}}$	M1 A2	**(10)**
4.	*(a)*	$2\sin x - 3\cos x = R\sin x\cos\alpha - R\cos x\sin\alpha$		
		$R\cos\alpha = 2,\ R\sin\alpha = 3$		
		$\therefore\ R = \sqrt{2^2 + 3^2} = \sqrt{13}$	M1 A1	
		$\tan\alpha = \frac{3}{2},\ \alpha = 56.3$ (3sf)	M1 A1	
		$\therefore\ 2\sin x° - 3\cos x° = \sqrt{13}\sin(x - 56.3)°$		
	(b)	$\operatorname{cosec} x° + 3\cot x° = 2 \ \Rightarrow\ \frac{1}{\sin x} + \frac{3\cos x}{\sin x} = 2$		
		$\Rightarrow\ 1 + 3\cos x = 2\sin x$		
		$\Rightarrow\ 2\sin x° - 3\cos x° = 1$	B1	
	(c)	$\sqrt{13}\sin(x - 56.31) = 1$		
		$\sin(x - 56.31) = \frac{1}{\sqrt{13}}$	M1	
		$x - 56.31 = 16.10,\ 180 - 16.10 = 16.10, 163.90$	B1 M1	
		$x = 72.4, 220.2$ (1dp)	A2	**(10)**

5.	*(a)*	let $\mathrm{f}(x) = 2x^3 - x^2 + 4x + 15$		
		$\mathrm{f}(-\frac{3}{2}) = -\frac{27}{4} - \frac{9}{4} - 6 + 15 = 0 \ \therefore\ (2x + 3)$ is a factor	M1 A1	
	(b)	$\begin{array}{r l} & x^2 - 2x + 5 \\ 2x+3 \,\big) & 2x^3 - x^2 + 4x + 15 \\ & 2x^3 + 3x^2 \\ & \quad - 4x^2 + 4x \\ & \quad - 4x^2 - 6x \\ & \qquad 10x + 15 \\ & \qquad 10x + 15 \end{array}$	M1 A1	
		$\therefore\ \mathrm{f}(x) = (2x + 3)(x^2 - 2x + 5)$		
		$\therefore\ \frac{2x^2+x-3}{2x^3-x^2+4x+15} = \frac{(2x+3)(x-1)}{(2x+3)(x^2-2x+5)} = \frac{x-1}{x^2-2x+5}$	M1 A1	
	(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1\times(x^2-2x+5)-(x-1)(2x-2)}{(x^2-2x+5)^2} = \frac{-x^2+2x+3}{(x^2-2x+5)^2}$	M1 A2	
		SP: $\frac{-x^2+2x+3}{(x^2-2x+5)^2} = 0$		
		$-x^2 + 2x + 3 = 0,\quad -(x + 1)(x - 3) = 0$	M1	
		$x = -1, 3 \quad \therefore\ (-1, -\frac{1}{4}), (3, \frac{1}{4})$	A2	**(12)**
6.	*(a)*	$P = 30 + 50\mathrm{e}^{0.002\times 30} = 83.1$	M1	
		$\therefore$ population = 83 100 (3sf)	A1	
	(b)	$30 + 50\mathrm{e}^{0.002t} > 84$	M1	
		$\mathrm{e}^{0.002t} > \frac{54}{50}$	A1	
		$t > \frac{1}{0.002}\ln\frac{54}{50},\ t > 38.5 \ \therefore\ 2018$	M1 A1	
	(c)	$30 + 50\mathrm{e}^{0.002t} = 26 + 50\mathrm{e}^{0.003t},\quad 50\mathrm{e}^{0.003t} - 50\mathrm{e}^{0.002t} = 4$		
		$\mathrm{e}^{0.003t} - \mathrm{e}^{0.002t} = 0.08,\quad \mathrm{e}^{0.002t}(\mathrm{e}^{0.001t} - 1) = 0.08$	M1	
		$\mathrm{e}^{0.001t} - 1 = 0.08\mathrm{e}^{-0.002t}$	M1	
		$0.001t = \ln(1 + 0.08\mathrm{e}^{-0.002t})$		
		$t = 1000\ln(1 + 0.08\mathrm{e}^{-0.002t})$	A1	
	(d)	$t_1 = 69.887,\ t_2 = 67.251,\ t_3 = 67.595$	M1 A2	
		$\therefore$ 2047	A1	**(13)**
7.	*(a)*	*(i)* 	M1 A2	
		(ii) 	M1 A2	
	(b)	$x = 0 \Rightarrow y = -1 \ \therefore\ b = -1$	B1	
		$y = 0 \Rightarrow 2 - \sqrt{x+9} = 0$		
		$x = 2^2 - 9 = -5 \ \therefore\ a = -5$	M1 A1	
	(c)	$y = 2 - \sqrt{x+9},\quad \sqrt{x+9} = 2 - y$		
		$x + 9 = (2 - y)^2$	M1	
		$x = (2 - y)^2 - 9$		
		$\therefore\ \mathrm{f}^{-1}(x) = (2 - x)^2 - 9$	M1 A1	
		$\mathrm{f}(-9) = 2 \ \therefore$ domain of $\mathrm{f}^{-1}(x)$ is $x \in \mathbb{R},\ x \le 2$	M1 A1	**(14)**
			Total	**(75)**