1. Solve the equation

$$3 \operatorname{cosec} \theta° + 8 \cos \theta° = 0$$

for θ in the interval $0 \leq \theta \leq 180$, giving your answers to 1 decimal place. **(6)**

---

2. The functions f and g are defined by

$$\mathrm{f} : x \rightarrow 1 - ax, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \rightarrow x^2 + 2ax + 2, \quad x \in \mathbb{R},$$

where a is a constant.

(a) Find the range of g in terms of a. **(3)**

Given that $\mathrm{gf}(3) = 7$,

(b) find the two possible values of a. **(4)**

---

3. *(a)* Solve the equation

$$\ln(3x + 1) = 2$$

giving your answer in terms of e. **(3)**

(b) Prove, by counter-example, that the statement

"$\ln(3x^2 + 5x + 3) \geq 0$ for all real values of x"

is false. **(5)**

---

4. A curve has the equation $x = y\sqrt{1-2y}$.

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{1-2y}}{1-3y}.$$ **(5)**

The point A on the curve has y-coordinate -1.

(b) Show that the equation of tangent to the curve at A can be written in the form

$$\sqrt{3}\,x + py + q = 0$$

where p and q are integers to be found. **(3)**

---

5. *(a)* Sketch the graph of $y = 2 + \sec\left(x - \frac{\pi}{6}\right)$ for x in the interval $0 \leq x \leq 2\pi$.

Show on your sketch the coordinates of any turning points and the equations of any asymptotes. **(5)**

(b) Find, in terms of π, the x-coordinates of the points where the graph crosses the x-axis. **(5)**

6.

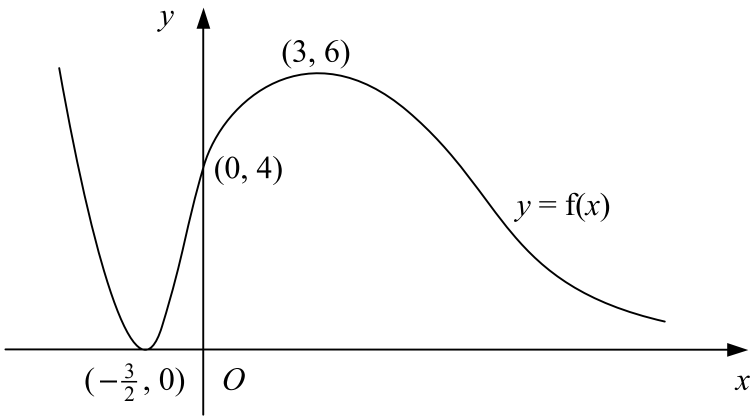

Figure 1

Figure 1 shows the curve $y = \mathrm{f}(x)$ which has a minimum point at $(-\frac{3}{2}, 0)$, a maximum point at $(3, 6)$ and crosses the y-axis at $(0, 4)$.

Sketch each of the following graphs on separate diagrams. In each case, show the coordinates of any turning points and of any points where the graph meets the coordinate axes.

(a) $y = \mathrm{f}(|x|)$ **(3)**

(b) $y = 2 + \mathrm{f}(x + 3)$ **(4)**

(c) $y = \frac{1}{2}\mathrm{f}(-x)$ **(4)**

Turn over

7. $$\mathrm{f}(x) = 1 + \frac{4x}{2x-5} - \frac{15}{2x^2-7x+5}, \quad x \in \mathbb{R}, \quad x < 1.$$

(a) Show that

$$\mathrm{f}(x) = \frac{3x+2}{x-1}.$$ **(5)**

(b) Find an expression for the inverse function $\mathrm{f}^{-1}(x)$ and state its domain. **(5)**

(c) Solve the equation $\mathrm{f}(x) = 2$. **(2)**

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8. A curve has the equation $y = x^2 - \sqrt{4 + \ln x}$.

(a) Show that the tangent to the curve at the point where $x = 1$ has the equation

$$7x - 4y = 11.$$ **(5)**

The curve has a stationary point with x-coordinate α.

(b) Show that $0.3 < \alpha < 0.4$ **(3)**

(c) Show that α is a solution of the equation

$$x = \tfrac{1}{2}(4 + \ln x)^{-\frac{1}{4}}.$$ **(2)**

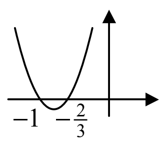
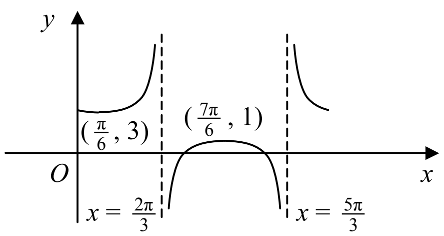
(d) Use the iteration formula

$$x_{n+1} = \tfrac{1}{2}(4 + \ln x_n)^{-\frac{1}{4}},$$

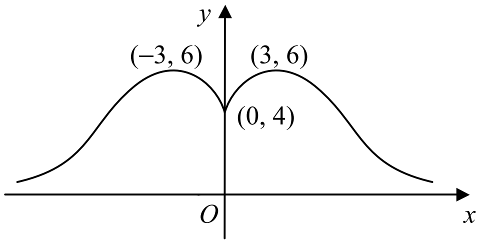
with $x_0 = 0.35$, to find x_1, x_2, x_3 and x_4, giving your answers to 5 decimal places. **(3)**

END

C3 Paper F – Marking Guide

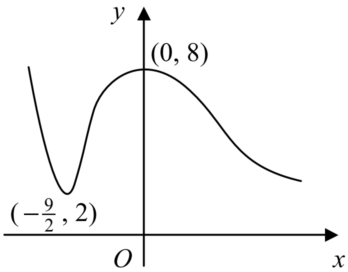
1.		$\frac{3}{\sin\theta} = -8\cos\theta$	M1	
		$3 = -8\sin\theta\cos\theta = -4\sin 2\theta$	M1	
		$\sin 2\theta = -\frac{3}{4}$	A1	
		$2\theta = 180 + 48.590,\ 360 - 48.590 = 228.590,\ 311.410$	M1	
		$\theta = 114.3,\ 155.7$ (1dp)	A2	**(6)**
2.	*(a)*	$g(x) = (x + a)^2 - a^2 + 2$	M1 A1	
		$\therefore\ g(x) \geq 2 - a^2$	A1	
	(b)	$gf(3) = g(1 - 3a) = (1 - 3a)^2 + 2a(1 - 3a) + 2$	M1	
		$\therefore\ 1 - 6a + 9a^2 + 2a - 6a^2 + 2 = 7, \quad 3a^2 - 4a - 4 = 0$	A1	
		$(3a + 2)(a - 2) = 0$	M1	
		$a = -\frac{2}{3},\ 2$	A1	**(7)**
3.	*(a)*	$3x + 1 = e^2$	M1	
		$x = \frac{1}{3}(e^2 - 1)$	M1 A1	
	(b)	consider $\ln(3x^2 + 5x + 3) \geq 0$		
		$\Rightarrow\ 3x^2 + 5x + 3 \geq 1$	M1	
		$3x^2 + 5x + 2 \geq 0$		
		$(3x + 2)(x + 1) \geq 0$ 	M1	
		$x \leq -1$ or $x \geq -\frac{2}{3}$	A1	
		$\therefore$ if (e.g.) $x = -\frac{3}{4}$, $\ln(3x^2 + 5x + 3) = \ln\frac{15}{16} = -0.0645...$	M1	
		$\therefore$ if $x = -\frac{3}{4}$, $\ln(3x^2 + 5x + 3) < 0 \quad \therefore$ statement is false	A1	**(8)**
4.	*(a)*	$\frac{dx}{dy} = 1 \times \sqrt{1-2y} + y \times \frac{1}{2}(1-2y)^{-\frac{1}{2}} \times (-2)$	M1 A1	
		$= \sqrt{1-2y} - \frac{y}{\sqrt{1-2y}} = \frac{(1-2y)-y}{\sqrt{1-2y}} = \frac{1-3y}{\sqrt{1-2y}}$	M1	
		$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{\sqrt{1-2y}}{1-3y}$	M1 A1	
	(b)	$y = -1,\ x = -\sqrt{3},\ \text{grad} = \frac{1}{4}\sqrt{3}$	B1	
		$\therefore\ y + 1 = \frac{1}{4}\sqrt{3}(x + \sqrt{3})$	M1	
		$4y + 4 = \sqrt{3}x + 3$		
		$\sqrt{3}x - 4y - 1 = 0 \qquad [\,p = -4,\ q = -1\,]$	A1	**(8)**
5.	*(a)*		M2 A3	
	(b)	$2 + \sec(x - \frac{\pi}{6}) = 0, \quad \sec(x - \frac{\pi}{6}) = -2, \quad \cos(x - \frac{\pi}{6}) = -\frac{1}{2}$	M1	
		$x - \frac{\pi}{6} = \pi - \frac{\pi}{3},\ \pi + \frac{\pi}{3} = \frac{2\pi}{3},\ \frac{4\pi}{3}$	B1 M1	
		$x = \frac{5\pi}{6},\ \frac{3\pi}{2}$	A2	**(10)**

6. *(a)*

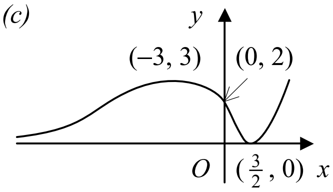


B3

(b)



(c)



M2 A2

M2 A2 **(11)**

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7. *(a)* $\mathrm{f}(x) = 1 + \frac{4x}{2x-5} - \frac{15}{(2x-5)(x-1)}$ B1

$= \frac{2x^2-7x+5+4x(x-1)-15}{(2x-5)(x-1)}$ M1 A1

$= \frac{6x^2-11x-10}{(2x-5)(x-1)} = \frac{(3x+2)(2x-5)}{(2x-5)(x-1)} = \frac{3x+2}{x-1}$ M1 A1

(b) $y = \frac{3x+2}{x-1}, \quad y(x-1) = 3x+2$ M1

$x(y-3) = y+2$ M1

$x = \frac{y+2}{y-3}$

$\therefore\ \mathrm{f}^{-1}(x) = \frac{x+2}{x-3}$ A1

$\mathrm{f}(x) = \frac{3(x-1)+5}{x-1} = 3 + \frac{5}{x-1}$ M1

$x < 1\ \therefore\ \mathrm{f}(x) < 3\ \therefore$ domain of $\mathrm{f}^{-1}(x)$ is $x \in \mathbb{R},\ x < 3$ A1

(c) $\mathrm{f}(x) = 2 \Rightarrow x = \mathrm{f}^{-1}(2) = -4$ M1 A1 **(12)**

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8. *(a)* $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - \frac{1}{2}(4+\ln x)^{-\frac{1}{2}} \times \frac{1}{x} = 2x - \frac{1}{2x\sqrt{4+\ln x}}$ M1 A1

$x = 1,\ y = -1,\ \text{grad} = \frac{7}{4}$ A1

$\therefore\ y + 1 = \frac{7}{4}(x-1)$ M1

$4y + 4 = 7x - 7$

$7x - 4y = 11$ A1

(b) SP: $2x - \frac{1}{2x\sqrt{4+\ln x}} = 0$ M1

let $\mathrm{f}(x) = 2x - \frac{1}{2x\sqrt{4+\ln x}}$, $\mathrm{f}(0.3) = -0.40,\ \mathrm{f}(0.4) = 0.088$ M1

sign change, f(x) continuous $\therefore$ root A1

(c) $2x - \frac{1}{2x\sqrt{4+\ln x}} = 0 \Rightarrow 2x = \frac{1}{2x\sqrt{4+\ln x}}$

$x^2 = \frac{1}{4\sqrt{4+\ln x}} = \frac{1}{4}(4+\ln x)^{-\frac{1}{2}}$ M1

$x = \sqrt{\frac{1}{4}(4+\ln x)^{-\frac{1}{2}}} = \frac{1}{2}(4+\ln x)^{-\frac{1}{4}}$ A1

(d) $x_1 = 0.38151,\ x_2 = 0.37877,\ x_3 = 0.37900,\ x_4 = 0.37898$ (5dp) M1 A2 **(13)**

Total **(75)**