1. Express

$$\frac{2x^3 + x^2}{x^2 - 4} \times \frac{x - 2}{2x^2 - 5x - 3}$$

as a single fraction in its simplest form. **(5)**

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2. *(a)* Prove that, for $\cos x \neq 0$,

$$\sin 2x - \tan x \equiv \tan x \cos 2x. \quad \textbf{(5)}$$

(b) Hence, or otherwise, solve the equation

$$\sin 2x - \tan x = 2 \cos 2x,$$

for x in the interval $0 \leq x \leq 180°$. **(5)**

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3. $$\mathrm{f}(x) = x^2 + 5x - 2 \sec x, \quad x \in \mathbb{R}, \quad -\tfrac{\pi}{2} < x < \tfrac{\pi}{2}.$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root in the interval $[1, 1.5]$. **(2)**

A more accurate estimate of this root is to be found using iterations of the form

$$x_{n+1} = \arccos \mathrm{g}(x_n).$$

(b) Find a suitable form for $\mathrm{g}(x)$ and use this formula with $x_0 = 1.25$ to find x_1, x_2, x_3 and x_4. Give the value of x_4 to 3 decimal places. **(6)**

The curve $y = \mathrm{f}(x)$ has a stationary point at P.

(c) Show that the x-coordinate of P is 1.0535 correct to 5 significant figures. **(3)**

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4. *(a)* Differentiate each of the following with respect to x and simplify your answers.

(i) $\sqrt{1 - \cos x}$

(ii) $x^3 \ln x$ **(6)**

(b) Given that

$$x = \frac{y + 1}{3 - 2y},$$

find and simplify an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y. **(5)**

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5. *(a)* Express $\sqrt{3}\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

(b) State the maximum value of $\sqrt{3}\sin\theta + \cos\theta$ and the smallest positive value of θ for which this maximum value occurs. **(3)**

(c) Solve the equation

$$\sqrt{3}\sin\theta + \cos\theta + \sqrt{3} = 0,$$

for θ in the interval $-\pi \le \theta \le \pi$, giving your answers in terms of π. **(5)**

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6. The function f is defined by

$$\mathrm{f}(x) \equiv 3 - x^2, \quad x \in \mathbb{R}, \quad x \ge 0.$$

(a) State the range of f. **(1)**

(b) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram. **(3)**

(c) Find an expression for $\mathrm{f}^{-1}(x)$ and state its domain. **(4)**

The function g is defined by

$$\mathrm{g}(x) \equiv \frac{8}{3 - x}, \quad x \in \mathbb{R}, \quad x \ne 3.$$

(d) Evaluate fg(–3). **(2)**

(e) Solve the equation

$$\mathrm{f}^{-1}(x) = \mathrm{g}(x).$$ **(3)**

Turn over

7.

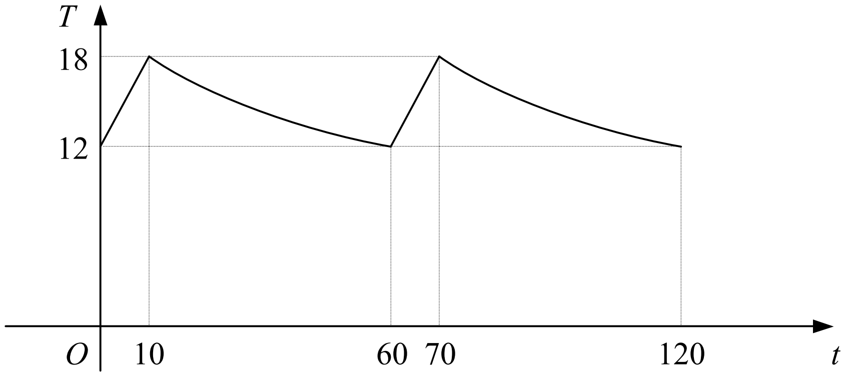

Figure 1

Figure 1 shows a graph of the temperature of a room, T °C, at time t minutes.

The temperature is controlled by a thermostat such that when the temperature falls to 12°C, a heater is turned on until the temperature reaches 18°C. The room then cools until the temperature again falls to 12°C.

For t in the interval $10 \leq t \leq 60$, T is given by

$$T = 5 + Ae^{-kt},$$

where A and k are constants.

Given that $T = 18$ when $t = 10$ and that $T = 12$ when $t = 60$,

(a) show that $k = 0.0124$ to 3 significant figures and find the value of A, **(6)**

(b) find the rate at which the temperature of the room is decreasing when $t = 20$. **(4)**

The temperature again reaches 18°C when $t = 70$ and the graph for $70 \leq t \leq 120$ is a translation of the graph for $10 \leq t \leq 60$.

(c) Find the value of the constant B such that for $70 \leq t \leq 120$

$$T = 5 + Be^{-kt}.$$ **(3)**

END

C3 Paper E – Marking Guide

1.	$= \dfrac{x^2(2x+1)}{(x+2)(x-2)} \times \dfrac{x-2}{(2x+1)(x-3)}$	M1 A2	
	$= \dfrac{x^2}{(x+2)(x-3)}$	M1 A1	**(5)**

2.	*(a)* LHS $\equiv 2\sin x\cos x - \dfrac{\sin x}{\cos x}$	M1	
	$\equiv \dfrac{2\sin x\cos^2 x - \sin x}{\cos x}$	M1 A1	
	$\equiv \dfrac{\sin x(2\cos^2 x - 1)}{\cos x} \equiv \dfrac{\sin x}{\cos x} \times \cos 2x \equiv \tan x\cos 2x \equiv$ RHS	M1 A1	
	(b) $\tan x\cos 2x = 2\cos 2x$		
	$\cos 2x(\tan x - 2) = 0$	M1	
	$\cos 2x = 0$ or $\tan x = 2$	A1	
	$2x = 90, 270$ or $x = 63.4$	B1	
	$x = 45°, 63.4°$ (1dp), $135°$	M1 A1	**(10)**

3.	*(a)* $f(1) = 2.30$, $f(1.5) = -18.5$	M1	
	sign change, $f(x)$ continuous $\therefore$ root	A1	
	(b) $x^2 + 5x - 2\sec x = 0 \;\Rightarrow\; x^2 + 5x = \dfrac{2}{\cos x}$	M1	
	$\cos x = \dfrac{2}{x^2+5x}$	M1	
	$x = \arccos\dfrac{2}{x^2+5x} \;\therefore\; g(x) = \dfrac{2}{x^2+5x}$	A1	
	$x_1 = 1.3119$, $x_2 = 1.3269$, $x_3 = 1.3302$, $x_4 = 1.3310 = 1.331$ (3dp)	M1 A2	
	(c) $f'(x) = 2x + 5 - 2\sec x\tan x$	M1	
	SP: $2x + 5 - 2\sec x\tan x = 0$		
	$f'(1.05345) = 0.00046$, $f'(1.05355) = -0.0022$	M1	
	sign change, $f'(x)$ continuous $\therefore$ root $\therefore$ x-coord of $P = 1.0535$ (5sf)	A1	**(11)**

4.	*(a)* *(i)* $= \tfrac{1}{2}(1-\cos x)^{-\frac{1}{2}} \times \sin x = \dfrac{\sin x}{2\sqrt{1-\cos x}}$	M1 A2	
	(ii) $= 3x^2 \times \ln x + x^3 \times \dfrac{1}{x} = x^2(3\ln x + 1)$	M1 A2	
	(b) $\dfrac{dx}{dy} = \dfrac{1\times(3-2y)-(y+1)\times(-2)}{(3-2y)^2} = \dfrac{5}{(3-2y)^2}$	M1 A2	
	$\dfrac{dy}{dx} = 1 \div \dfrac{dx}{dy} = \tfrac{1}{5}(3-2y)^2$	M1 A1	**(11)**

5.	*(a)* $\sqrt{3}\sin\theta + \cos\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$		
	$R\cos\alpha = \sqrt{3}$, $R\sin\alpha = 1 \quad \therefore\; R = \sqrt{3+1} = 2$	M1 A1	
	$\tan\alpha = \dfrac{1}{\sqrt{3}}$, $\alpha = \tfrac{\pi}{6} \quad \therefore\; \sqrt{3}\sin\theta + \cos\theta = 2\sin(\theta + \tfrac{\pi}{6})$	M1 A1	
	(b) maximum $= 2$	B1	
	occurs when $\theta + \tfrac{\pi}{6} = \tfrac{\pi}{2}$, $\theta = \tfrac{\pi}{3}$	M1 A1	
	(c) $2\sin(\theta + \tfrac{\pi}{6}) + \sqrt{3} = 0$, $\sin(\theta + \tfrac{\pi}{6}) = -\dfrac{\sqrt{3}}{2}$	M1	
	$\theta + \tfrac{\pi}{6} = -\tfrac{\pi}{3}, -\pi + \tfrac{\pi}{3} = -\tfrac{\pi}{3}, -\tfrac{2\pi}{3}$	B1 M1	
	$\theta = -\tfrac{5\pi}{6}, -\tfrac{\pi}{2}$	A2	**(12)**

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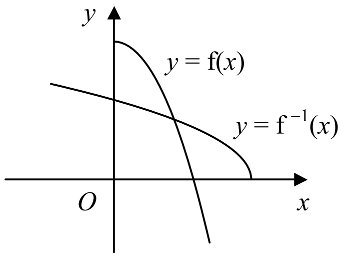
6. *(a)* $f(x) \leq 3$ B1

(b)



B3

(c) $y = 3 - x^2$

$x^2 = 3 - y$ M1

$x = \pm\sqrt{3-y}$

$f^{-1}(x) = \sqrt{3-x}\,,\ x \in \mathbb{R},\ x \leq 3$ M1 A2

(d) $= f(\frac{4}{3}) = \frac{11}{9}$ M1 A1

(e) $\sqrt{3-x} = \frac{8}{3-x}$

$3 - x = \frac{64}{(3-x)^2}$ M1

$(3 - x)^3 = 64$

$3 - x = 4$ M1

$x = -1$ A1 **(13)**

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7. *(a)* $t = 10, T = 18 \quad \Rightarrow \quad 18 = 5 + Ae^{-10k} \quad (1)$

$t = 60, T = 12 \quad \Rightarrow \quad 12 = 5 + Ae^{-60k} \quad (2)$ M1

$(1) \quad \Rightarrow \quad A = \frac{13}{e^{-10k}} = 13e^{10k}$ M1

sub (2) $\quad \Rightarrow \quad 7 = 13e^{10k} \times e^{-60k}$

$e^{-50k} = \frac{7}{13}$ A1

$\therefore\ k = -\frac{1}{50}\ln\frac{7}{13} = 0.0124$ (3sf) M1 A1

$\therefore\ A = 13e^{10 \times 0.01238} = 14.7$ (3sf) A1

(b) $T = 5 + 14.71e^{-0.01238t}$

$\frac{dT}{dt} = -0.01238 \times 14.71\ e^{-0.01238t} = -0.1822e^{-0.01238t}$ M1 A1

when $t = 20$, $\frac{dT}{dt} = -0.1822e^{-0.01238 \times 20} = -0.142$ M1

$\therefore$ temperature decreasing at rate of 0.142 °C per minute (3sf) A1

(c) $T = 5 + 14.71e^{-0.01238(t-60)}$ M1

$= 5 + 14.71e^{0.7428 - 0.01238t}$

$= 5 + 14.71e^{0.7428} \times e^{-0.01238t}$ M1

$= 5 + 30.9e^{-0.01238t}, \quad B = 30.9$ (3sf) A1 **(13)**

Total **(75)**