1. The function f is defined by

$$f(x) \equiv 2 + \ln(3x - 2), \quad x \in \mathbb{R}, \quad x > \tfrac{2}{3}.$$

(a) Find the exact value of ff(1). **(2)**

(b) Find an expression for $f^{-1}(x)$. **(3)**

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2. Find, to 2 decimal places, the solutions of the equation

$$3\cot^2 x - 4\operatorname{cosec} x + \operatorname{cosec}^2 x = 0$$

in the interval $0 \leq x \leq 2\pi$. **(6)**

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3. *(a)* Given that $y = \ln x$, find expressions in terms of y for

(i) $\log_2 x$,

(ii) $\ln \frac{x^2}{e}$. **(4)**

(b) Hence, or otherwise, solve the equation

$$\log_2 x = 4 - \ln \frac{x^2}{e},$$

giving your answer to 2 decimal places. **(4)**

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4. *(a)* Use the identities for $(\sin A + \sin B)$ and $(\cos A + \cos B)$ to prove that

$$\frac{\sin 2x + \sin 2y}{\cos 2x + \cos 2y} \equiv \tan(x + y).$$ **(4)**

(b) Hence, show that

$$\tan 52.5° = \sqrt{6} - \sqrt{3} - \sqrt{2} + 2.$$ **(5)**

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5. $$\mathrm{f}(x) = 3 - \frac{x-1}{x-3} + \frac{x+11}{2x^2-5x-3}, \quad x \in \mathbb{R}, \quad x < -1.$$

(a) Show that

$$\mathrm{f}(x) = \frac{4x-1}{2x+1}. \qquad \textbf{(5)}$$

(b) Find an equation for the tangent to the curve $y = \mathrm{f}(x)$ at the point where $x = -2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

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6. A curve has the equation $y = \mathrm{e}^{3x} \cos 2x$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **(2)**

(b) Show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^{3x}(5 \cos 2x - 12 \sin 2x)$. **(3)**

The curve has a stationary point in the interval $[0, 1]$.

(c) Find the x-coordinate of the stationary point to 3 significant figures. **(4)**

(d) Determine whether the stationary point is a maximum or minimum point and justify your answer. **(2)**

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7. *(a)* Sketch on the same diagram the graphs of $y = 4a^2 - x^2$ and $y = |2x - a|$, where a is a positive constant. Show, in terms of a, the coordinates of any points where each graph meets the coordinate axes. **(6)**

(b) Find the exact solutions of the equation

$$4 - x^2 = |2x - 1|. \qquad \textbf{(6)}$$

Turn over

8.

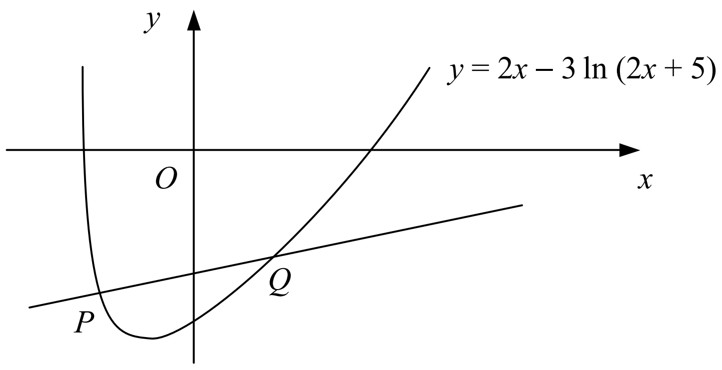

Figure 1

Figure 1 shows the curve with equation $y = 2x - 3\ln(2x + 5)$ and the normal to the curve at the point $P(-2, -4)$.

(a) Find an equation for the normal to the curve at P. **(4)**

The normal to the curve at P intersects the curve again at the point Q with x-coordinate q.

(b) Show that $1 < q < 2$. **(3)**

(c) Show that q is a solution of the equation

$$x = \tfrac{12}{7}\ln(2x + 5) - 2.$$ **(2)**

(d) Use the iterative formula

$$x_{n+1} = \tfrac{12}{7}\ln(2x_n + 5) - 2,$$

with $x_0 = 1.5$, to find the value of q to 3 significant figures and justify the accuracy of your answer. **(5)**

END

C3 Paper D – Marking Guide

1.	*(a)*	$= f(2) = 2 + \ln 4$	M1 A1	
	(b)	$y = 2 + \ln(3x - 2), \quad 3x - 2 = e^{y-2}$	M1	
		$x = \frac{1}{3}(2 + e^{y-2})$		
		$f^{-1}(x) = \frac{1}{3}(2 + e^{x-2})$	M1 A1	**(5)**

2.	$3(\text{cosec}^2 x - 1) - 4\,\text{cosec}\, x + \text{cosec}^2 x = 0$	M1	
	$4\,\text{cosec}^2 x - 4\,\text{cosec}\, x - 3 = 0$		
	$(2\,\text{cosec}\, x + 1)(2\,\text{cosec}\, x - 3) = 0$	M1	
	$\text{cosec}\, x = -\frac{1}{2}$ or $\frac{3}{2}$	A1	
	$\sin x = -2$ (no solutions) or $\frac{2}{3}$	M1	
	$x = 0.73,\ \pi - 0.7297$		
	$x = 0.73,\ 2.41$ (2dp)	A2	**(6)**

3.	*(a)*	*(i)* $= \frac{\ln x}{\ln 2} = \frac{y}{\ln 2}$	M1 A1	
		(ii) $= \ln x^2 - \ln e = 2\ln x - 1 = 2y - 1$	M1 A1	
	(b)	$\frac{y}{\ln 2} = 4 - (2y - 1)$	M1	
		$y = (5 - 2y)\ln 2$		
		$y(2\ln 2 + 1) = 5\ln 2$	M1	
		$y = \frac{5\ln 2}{2\ln 2 + 1}$	A1	
		$x = e^y = 4.27$ (2dp)	A1	**(8)**

4.	*(a)*	$\text{LHS} \equiv \frac{2\sin(x+y)\cos(x-y)}{2\cos(x+y)\cos(x-y)}$	M1 A1	
		$\equiv \frac{\sin(x+y)}{\cos(x+y)} \equiv \tan(x+y) \equiv \text{RHS}$	M1 A1	
	(b)	let $x = 30°,\ y = 22.5°\ \therefore\ \tan(30 + 22.5) = \frac{\sin 60 + \sin 45}{\cos 60 + \cos 45}$	M1	
		$\tan 52.5 = \frac{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}{\frac{1}{2} + \frac{1}{\sqrt{2}}} = \frac{\sqrt{3} + \sqrt{2}}{1 + \sqrt{2}}$	B1 A1	
		$= \frac{\sqrt{3} + \sqrt{2}}{1 + \sqrt{2}} \times \frac{1 - \sqrt{2}}{1 - \sqrt{2}}$	M1	
		$= \frac{\sqrt{3} - \sqrt{6} + \sqrt{2} - 2}{1 - 2} = \sqrt{6} - \sqrt{3} - \sqrt{2} + 2$	A1	**(9)**

5.	*(a)*	$f(x) = 3 - \frac{x-1}{x-3} + \frac{x+11}{(2x+1)(x-3)}$	B1	
		$= \frac{3(2x^2 - 5x - 3) - (x-1)(2x+1) + (x+11)}{(2x+1)(x-3)}$	M1 A1	
		$= \frac{4x^2 - 13x + 3}{(2x+1)(x-3)} = \frac{(4x-1)(x-3)}{(2x+1)(x-3)} = \frac{4x-1}{2x+1}$	M1 A1	
	(b)	$f'(x) = \frac{4 \times (2x+1) - (4x-1) \times 2}{(2x+1)^2} = \frac{6}{(2x+1)^2}$	M1 A1	
		$x = -2 \Rightarrow y = 3,$ grad $= \frac{2}{3}$	A1	
		$\therefore\ y - 3 = \frac{2}{3}(x + 2)$	M1	
		$3y - 9 = 2x + 4$		
		$2x - 3y + 13 = 0$	A1	**(10)**

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6. *(a)* $\frac{dy}{dx} = 3e^{3x} \times \cos 2x + e^{3x} \times (-2\sin 2x) = e^{3x}(3\cos 2x - 2\sin 2x)$ M1 A1

(b) $\frac{d^2y}{dx^2} = 3e^{3x} \times (3\cos 2x - 2\sin 2x) + e^{3x}(-6\sin 2x - 4\cos 2x)$ M1 A1

$= e^{3x}(5\cos 2x - 12\sin 2x)$ A1

(c) SP: $e^{3x}(3\cos 2x - 2\sin 2x) = 0$

$3\cos 2x = 2\sin 2x$ M1

$\tan 2x = \frac{3}{2}$ M1

$2x = 0.98279$, $x = 0.491$ (3sf) M1 A1

(d) when $x = 0.491$, $\frac{d^2y}{dx^2} = -31.5$, $\frac{d^2y}{dx^2} < 0$ $\therefore$ maximum M1 A1 **(11)**

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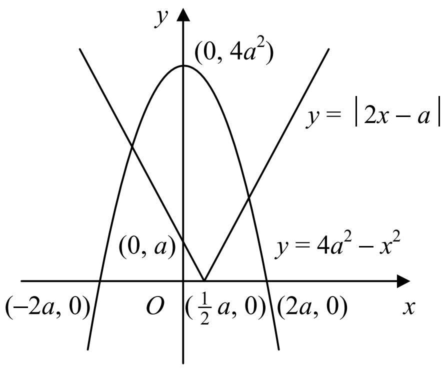
7. *(a)*



B3

B3

(b) $4 - x^2 = 2x - 1$ M1

$x^2 + 2x - 5 = 0$, $x = \frac{-2 \pm \sqrt{4+20}}{2} = \frac{-2 \pm 2\sqrt{6}}{2}$ M1

$x > \frac{1}{2}$ $\therefore x = -1 + \sqrt{6}$ A1

$4 - x^2 = -(2x - 1)$ M1

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$ M1

$x < \frac{1}{2}$ $\therefore x = -1$, $x = -1, -1 + \sqrt{6}$ A1 **(12)**

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8. *(a)* $\frac{dy}{dx} = 2 - \frac{3}{2x+5} \times 2 = 2 - \frac{6}{2x+5}$ M1

grad $= -4$, grad of normal $= \frac{1}{4}$ A1

$\therefore$ $y + 4 = \frac{1}{4}(x + 2)$ $[\, y = \frac{1}{4}x - \frac{7}{2} \,]$ M1 A1

(b) $\frac{1}{4}x - \frac{7}{2} = 2x - 3\ln(2x + 5)$ M1

$\frac{7}{4}x + \frac{7}{2} - 3\ln(2x + 5) = 0$, let $f(x) = \frac{7}{4}x + \frac{7}{2} - 3\ln(2x + 5)$

$f(1) = -0.59$, $f(2) = 0.41$ M1

sign change, $f(x)$ continuous $\therefore$ root A1

(c) $\frac{7}{4}x + \frac{7}{2} - 3\ln(2x + 5) = 0$

$7x + 14 - 12\ln(2x + 5) = 0$

$7x = 12\ln(2x + 5) - 14$ M1

$x = \frac{12}{7}\ln(2x + 5) - 2$ A1

(d) $x_1 = 1.5648$, $x_2 = 1.5923$, $x_3 = 1.6039$, $x_4 = 1.6087$, $x_5 = 1.6107$ M1 A1

$q = 1.61$ (3sf) A1

$f(1.605) = -0.0073$, $f(1.615) = 0.0029$ M1

sign change, $f(x)$ continuous $\therefore$ root $\therefore q = 1.61$ (3sf) A1 **(14)**

Total **(75)**