1. *(a)* Express

$$\frac{x+4}{2x^2+3x+1} - \frac{2}{2x+1}$$

as a single fraction in its simplest form. **(3)**

(b) Hence, find the values of x such that

$$\frac{x+4}{2x^2+3x+1} - \frac{2}{2x+1} = \frac{1}{2}.$$ **(3)**

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2. *(a)* Prove, by counter-example, that the statement

"cosec θ – sin θ > 0 for all values of θ in the interval $0 < \theta < \pi$"

is false. **(2)**

(b) Find the values of θ in the interval $0 < \theta < \pi$ such that

$$\operatorname{cosec}\theta - \sin\theta = 2,$$

giving your answers to 2 decimal places. **(5)**

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3. Solve each equation, giving your answers in exact form.

(a) $\ln(2x - 3) = 1$ **(3)**

(b) $3e^{y} + 5e^{-y} = 16$ **(5)**

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4. Differentiate each of the following with respect to x and simplify your answers.

(a) $\ln(3x - 2)$ **(2)**

(b) $\dfrac{2x+1}{1-x}$ **(3)**

(c) $x^{\frac{3}{2}}e^{2x}$ **(3)**

5.

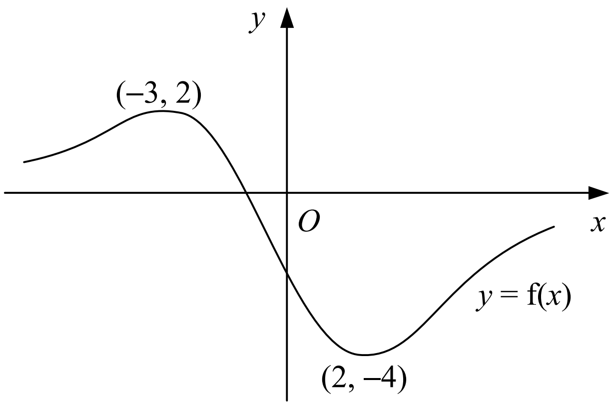

Figure 1

Figure 1 shows the curve $y = \mathrm{f}(x)$ which has a maximum point at $(-3, 2)$ and a minimum point at $(2, -4)$.

(a) Showing the coordinates of any stationary points, sketch on separate diagrams the graphs of

(i) $y = \mathrm{f}(|x|)$,

(ii) $y = 3\mathrm{f}(2x)$. **(7)**

(b) Write down the values of the constants a and b such that the curve with equation $y = a + \mathrm{f}(x + b)$ has a minimum point at the origin O. **(2)**

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6. The function f is defined by

$$\mathrm{f}(x) \equiv 4 - \ln 3x, \quad x \in \mathbb{R}, \quad x > 0.$$

(a) Solve the equation $\mathrm{f}(x) = 0$. **(2)**

(b) Sketch the curve $y = \mathrm{f}(x)$. **(2)**

(c) Find an expression for the inverse function, $\mathrm{f}^{-1}(x)$. **(3)**

The function g is defined by

$$\mathrm{g}(x) \equiv \mathrm{e}^{2-x}, \quad x \in \mathbb{R}.$$

(d) Show that

$$\mathrm{fg}(x) = x + a - \ln b,$$

where a and b are integers to be found. **(3)**

Turn over

7. *(a)* Express $4\sin x + 3\cos x$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

(b) State the minimum value of $4\sin x + 3\cos x$ and the smallest positive value of x for which this minimum value occurs. **(3)**

(c) Solve the equation

$$4\sin 2\theta + 3\cos 2\theta = 2,$$

for θ in the interval $0 \leq \theta \leq \pi$, giving your answers to 2 decimal places. **(6)**

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8. The curve C has the equation $y = \sqrt{x} + e^{1-4x}$, $x \geq 0$.

(a) Find an equation for the normal to the curve at the point $(\frac{1}{4}, \frac{3}{2})$. **(4)**

The curve C has a stationary point with x-coordinate α where $0.5 < \alpha < 1$.

(b) Show that α is a solution of the equation

$$x = \tfrac{1}{4}[1 + \ln(8\sqrt{x})].$$ **(3)**

(c) Use the iteration formula

$$x_{n+1} = \tfrac{1}{4}[1 + \ln(8\sqrt{x_n})],$$

with $x_0 = 1$ to find x_1, x_2, x_3 and x_4, giving the value of x_4 to 3 decimal places. **(3)**

(d) Show that your value for x_4 is the value of α correct to 3 decimal places. **(2)**

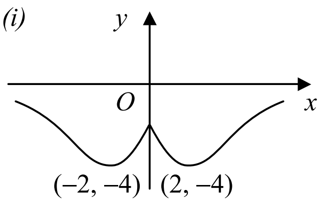
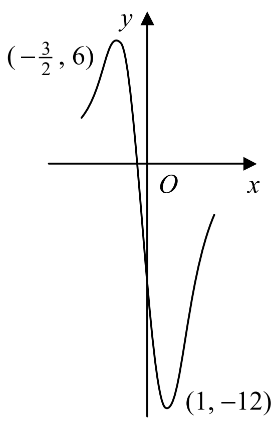
(e) Another attempt to find α is made using the iteration formula

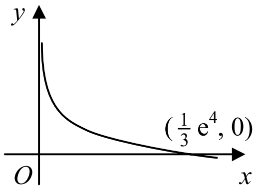
$$x_{n+1} = \tfrac{1}{64}e^{8x_n - 2},$$

with $x_0 = 1$. Describe the outcome of this attempt. **(2)**

END

C3 Paper C – Marking Guide

1.	*(a)*	$= \frac{x+4}{(2x+1)(x+1)} - \frac{2}{2x+1}$	M1	
		$= \frac{(x+4)-2(x+1)}{(2x+1)(x+1)}$	M1	
		$= \frac{2-x}{(2x+1)(x+1)}$	A1	
	(b)	$\frac{2-x}{(2x+1)(x+1)} = \frac{1}{2}$		
		$2(2 - x) = 2x^2 + 3x + 1$	M1	
		$2x^2 + 5x - 3 = 0$		
		$(2x - 1)(x + 3) = 0$	M1	
		$x = -3,\ \frac{1}{2}$	A1	**(6)**
2.	*(a)*	if $\theta = \frac{\pi}{2}$, $\sin\theta = 1$, $\operatorname{cosec}\theta = 1$	M1	
		$\therefore\ \operatorname{cosec}\theta - \sin\theta = 1 - 1 = 0$		
		$\therefore$ statement is false	A1	
	(b)	$1 - \sin^2\theta = 2\sin\theta$	M1	
		$\sin^2\theta + 2\sin\theta - 1 = 0$		
		$\sin\theta = \frac{-2\pm\sqrt{4+4}}{2} = -1 - \sqrt{2}$ (no solutions) or $-1 + \sqrt{2}$	M1 A1	
		$\theta = 0.4271,\ \pi - 0.4271$	M1	
		$\theta = 0.43,\ 2.71$ (2dp)	A1	**(7)**
3.	*(a)*	$2x - 3 = \mathrm{e}$	M1	
		$x = \frac{1}{2}(\mathrm{e} + 3)$	M1 A1	
	(b)	$3\mathrm{e}^{2y} - 16\mathrm{e}^{y} + 5 = 0$	M1	
		$(3\mathrm{e}^{y} - 1)(\mathrm{e}^{y} - 5) = 0$	M1	
		$\mathrm{e}^{y} = \frac{1}{3},\ 5$	A1	
		$y = \ln\frac{1}{3},\ \ln 5$	M1 A1	**(8)**
4.	*(a)*	$= \frac{1}{3x-2} \times 3 = \frac{3}{3x-2}$	M1 A1	
	(b)	$= \frac{2\times(1-x)-(2x+1)\times(-1)}{(1-x)^2} = \frac{3}{(1-x)^2}$	M1 A2	
	(c)	$= \frac{3}{2}x^{\frac{1}{2}} \times \mathrm{e}^{2x} + x^{\frac{3}{2}} \times 2\mathrm{e}^{2x} = \frac{1}{2}x^{\frac{1}{2}}\mathrm{e}^{2x}(3 + 4x)$	M1 A2	**(8)**
5.	*(a)*	*(i)*  *(ii)* 	B3 M2 A2	
	(b)	$a = 4,\ b = 2$	B2	**(9)**

6.	*(a)*	$4 - \ln 3x = 0, \quad \ln 3x = 4, \quad x = \frac{1}{3}e^4$	M1 A1	
	(b)		B2	
	(c)	$y = 4 - \ln 3x$		
		$\ln 3x = 4 - y$	M1	
		$x = \frac{1}{3}e^{4-y}$	M1	
		$\therefore\ f^{-1}(x) = \frac{1}{3}e^{4-x}$	A1	
	(d)	$fg(x) = 4 - \ln 3e^{2-x}$		
		$= 4 - (\ln 3 + \ln e^{2-x})$	M1	
		$= 4 - \ln 3 - (2 - x)$	M1	
		$= x + 2 - \ln 3 \qquad [a = 2, b = 3]$	A1	**(10)**
7.	*(a)*	$4 \sin x + 3 \cos x = R \sin x \cos \alpha + R \cos x \sin \alpha$		
		$R \cos \alpha = 4, \ R \sin \alpha = 3$		
		$\therefore\ R = \sqrt{4^2 + 3^2} = 5$	M1 A1	
		$\tan \alpha = \frac{3}{4}, \ \alpha = 0.644$ (3sf)	M1 A1	
		$\therefore\ 4 \sin x + 3 \cos x = 5 \sin (x + 0.644)$		
	(b)	minimum $= -5$	B1	
		occurs when $x + 0.6435 = \frac{3\pi}{2}, \ x = 4.07$ (3sf)	M1 A1	
	(c)	$5 \sin (2\theta + 0.6435) = 2$		
		$\sin (2\theta + 0.6435) = 0.4$	M1	
		$2\theta + 0.6435 = \pi - 0.4115, 2\pi + 0.4115$	B1 M1	
		$2\theta = 2.087, 6.051$	M1	
		$\theta = 1.04, 3.03$ (2dp)	A2	**(13)**
8.	*(a)*	$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 4e^{1-4x}$	M1	
		grad $= -3$, grad of normal $= \frac{1}{3}$	A1	
		$\therefore\ y - \frac{3}{2} = \frac{1}{3}(x - \frac{1}{4}) \qquad [\ 4x - 12y + 17 = 0\ ]$	M1 A1	
	(b)	SP: $\frac{1}{2}x^{-\frac{1}{2}} - 4e^{1-4x} = 0, \qquad \frac{1}{2\sqrt{x}} = 4e^{1-4x}$		
		$\frac{1}{8\sqrt{x}} = e^{1-4x}$	M1	
		$8\sqrt{x} = e^{4x-1}$		
		$4x - 1 = \ln 8\sqrt{x}$	M1	
		$x = \frac{1}{4}(1 + \ln 8\sqrt{x}\,)$	A1	
	(c)	$x_1 = 0.7699, x_2 = 0.7372, x_3 = 0.7317, x_4 = 0.7308 = 0.731$ (3dp)	M1 A2	
	(d)	let $f(x) = \frac{1}{2}x^{-\frac{1}{2}} - 4e^{1-4x}$		
		$f(0.7305) = -0.00025, \ f(0.7315) = 0.0017$	M1	
		sign change, $f(x)$ continuous $\therefore$ root	A1	
	(e)	$x_1 = 6.304, x_2 = 1.683 \times 10^{19}$		
		diverges rapidly away from root	B2	**(14)**
			Total	**(75)**