1. *(a)* Simplify

$$\frac{x^2 + 7x + 12}{2x^2 + 9x + 4}.$$ **(3)**

(b) Solve the equation

$$\ln (x^2 + 7x + 12) - 1 = \ln (2x^2 + 9x + 4),$$

giving your answer in terms of e. **(4)**

---

2. A curve has the equation $y = \sqrt{3x + 11}$.

The point P on the curve has x-coordinate 3.

(a) Show that the tangent to the curve at P has the equation

$$3x - 4\sqrt{5}\,y + 31 = 0.$$ **(6)**

The normal to the curve at P crosses the y-axis at Q.

(b) Find the y-coordinate of Q in the form $k\sqrt{5}$. **(3)**

---

3. *(a)* Use the identities for $\sin (A + B)$ and $\sin (A - B)$ to prove that

$$\sin P + \sin Q \equiv 2 \sin \frac{P+Q}{2} \cos \frac{P-Q}{2}.$$ **(4)**

(b) Find, in terms of π, the solutions of the equation

$$\sin 5x + \sin x = 0,$$

for x in the interval $0 \le x < \pi$. **(5)**

---

4. The curve with equation $y = x^{\frac{5}{2}} \ln \frac{x}{4}$, $x > 0$ crosses the x-axis at the point P.

(a) Write down the coordinates of P. **(1)**

The normal to the curve at P crosses the y-axis at the point Q.

(b) Find the area of triangle OPQ where O is the origin. **(6)**

The curve has a stationary point at R.

(c) Find the x-coordinate of R in exact form. **(3)**

---

5. $$f(x) \equiv 2x^2 + 4x + 2, \quad x \in \mathbb{R}, \quad x \geq -1.$$

(a) Express f(x) in the form $a(x + b)^2 + c$. **(2)**

(b) Describe fully two transformations that would map the graph of $y = x^2$, $x \geq 0$ onto the graph of $y = f(x)$. **(3)**

(c) Find an expression for $f^{-1}(x)$ and state its domain. **(4)**

(d) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram and state the relationship between them. **(4)**

---

6. $$f(x) = e^{3x + 1} - 2, \quad x \in \mathbb{R}.$$

(a) State the range of f. **(1)**

The curve $y = f(x)$ meets the y-axis at the point P and the x-axis at the point Q.

(b) Find the exact coordinates of P and Q. **(4)**

(c) Show that the tangent to the curve at P has the equation

$$y = 3\mathrm{e}x + \mathrm{e} - 2.$$ **(4)**

(d) Find to 3 significant figures the x-coordinate of the point where the tangent to the curve at P meets the tangent to the curve at Q. **(4)**

Turn over

7. *(a)* Solve the equation

$$\pi - 3 \arccos \theta = 0.$$ **(2)**

(b) Sketch on the same diagram the curves $y = \arccos (x - 1)$, $0 \leq x \leq 2$ and $y = \sqrt{x+2}$, $x \geq -2$. **(5)**

Given that α is the root of the equation

$$\arccos (x - 1) = \sqrt{x+2},$$

(c) show that $0 < \alpha < 1$, **(3)**

(d) use the iterative formula

$$x_{n+1} = 1 + \cos \sqrt{x_n + 2}$$

with $x_0 = 1$ to find α correct to 3 decimal places. **(4)**

END

C3 Paper B – Marking Guide

1. *(a)* $= \frac{(x+3)(x+4)}{(2x+1)(x+4)} = \frac{x+3}{2x+1}$ M1 A2

(b) $\ln (x^2 + 7x + 12) - \ln (2x^2 + 9x + 4) = 1, \quad \ln \frac{x^2+7x+12}{2x^2+9x+4} = 1$ M1

$\ln \frac{x+3}{2x+1} = 1, \quad \frac{x+3}{2x+1} = \mathrm{e}$ A1

$x + 3 = \mathrm{e}(2x + 1), \quad 3 - \mathrm{e} = x(2\mathrm{e} - 1)$ M1

$x = \frac{3-\mathrm{e}}{2\mathrm{e}-1}$ A1 **(7)**

---

2. *(a)* $x = 3, \ y = \sqrt{20} = 2\sqrt{5}$ B1

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(3x+11)^{-\frac{1}{2}} \times 3 = \frac{3}{2}(3x+11)^{-\frac{1}{2}}$ M1 A1

$\text{grad} = \frac{3}{4\sqrt{5}}$ A1

$\therefore \ y - 2\sqrt{5} = \frac{3}{4\sqrt{5}}(x - 3)$ M1

$4\sqrt{5}\,y - 40 = 3x - 9$

$3x - 4\sqrt{5}\,y + 31 = 0$ A1

(b) normal: $y - 2\sqrt{5} = -\frac{4\sqrt{5}}{3}(x - 3)$ M1

at Q, $x = 0 \ \therefore \ y - 2\sqrt{5} = 4\sqrt{5}, \quad y = 6\sqrt{5}$ M1 A1 **(9)**

---

3. *(a)* $\sin (A + B) \equiv \sin A \cos B + \cos A \sin B$

$\sin (A - B) \equiv \sin A \cos B - \cos A \sin B$

adding, $\sin (A + B) + \sin (A - B) \equiv 2 \sin A \cos B$ M1 A1

let $P = A + B, \ Q = A - B$

adding, $P + Q = 2A \ \Rightarrow \ A = \frac{P+Q}{2}$ M1

subtracting, $P - Q = 2B \ \Rightarrow \ B = \frac{P-Q}{2}$

$\therefore \ \sin P + \sin Q \equiv 2 \sin \frac{P+Q}{2} \cos \frac{P-Q}{2}$ A1

(b) $2 \sin 3x \cos 2x = 0$ M1

$\sin 3x = 0$ or $\cos 2x = 0$ A1

$3x = 0, \pi, 2\pi$ or $2x = \frac{\pi}{2}, \frac{3\pi}{2}$ M1

$x = 0, \frac{\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{4}$ A2 **(9)**

---

4. *(a)* $(4, 0)$ B1

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5}{2}x^{\frac{3}{2}} \times \ln \frac{x}{4} + x^{\frac{5}{2}} \times \frac{1}{x} = \frac{1}{2}x^{\frac{3}{2}}(5 \ln \frac{x}{4} + 2)$ M1 A1

grad $= 8$, grad of normal $= -\frac{1}{8}$ A1

$\therefore \ y - 0 = -\frac{1}{8}(x - 4)$ M1

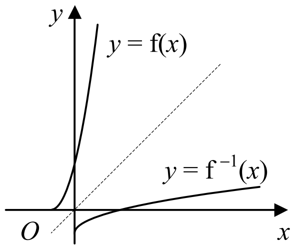
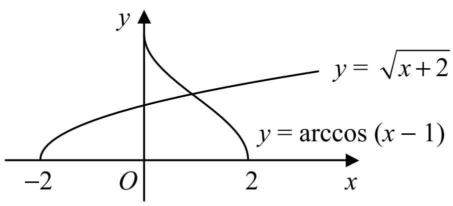
at Q, $x = 0, \ y = \frac{1}{2}$ A1

area $= \frac{1}{2} \times \frac{1}{2} \times 4 = 1$ A1

(c) $\frac{1}{2}x^{\frac{3}{2}}(5 \ln \frac{x}{4} + 2) = 0$

$\ln \frac{x}{4} = -\frac{2}{5}$ M1

$x = 4\mathrm{e}^{-\frac{2}{5}}$ M1 A1 **(10)**

5.	*(a)*	$= 2[x^2 + 2x] + 2 = 2[(x+1)^2 - 1] + 2$	M1	
		$= 2(x+1)^2$	A1	
	(b)	translation by 1 unit in negative x direction		
		stretch by scale factor of 2 in y direction (either first)	B3	
	(c)	$y = 2(x+1)^2, \quad \frac{y}{2} = (x+1)^2$	M1	
		$x + 1 = \pm\sqrt{\frac{y}{2}}$	M1	
		$x = -1 \pm \sqrt{\frac{y}{2}}$ (domain $\Rightarrow$ +), $\therefore$ $\mathrm{f}^{-1}(x) = -1 + \sqrt{\frac{x}{2}}$, $x \in \mathbb{R}$, $x \geq 0$	A2	
	(d)		B3	
		$y = \mathrm{f}^{-1}(x)$ is reflection of $y = \mathrm{f}(x)$ in line $y = x$	B1	**(13)**
6.	*(a)*	$\mathrm{f}(x) > -2$	B1	
	(b)	$x = 0,\ y = \mathrm{e} - 2 \quad \therefore\ P\,(0, \mathrm{e} - 2)$	B1	
		$y = 0,\ 0 = \mathrm{e}^{3x+1} - 2$	M1	
		$3x + 1 = \ln 2$	M1	
		$x = \frac{1}{3}(\ln 2 - 1) \quad \therefore\ Q\,(\frac{1}{3}(\ln 2 - 1), 0)$	A1	
	(c)	$\mathrm{f}'(x) = 3\mathrm{e}^{3x+1}$	M1	
		at P, grad $= 3\mathrm{e}$	A1	
		$\therefore\ y - (\mathrm{e} - 2) = 3\mathrm{e}(x - 0)$	M1	
		$y = 3\mathrm{e}x + \mathrm{e} - 2$	A1	
	(d)	at Q, grad $= 6$	B1	
		tangent at Q: $y - 0 = 6(x - \frac{1}{3}(\ln 2 - 1))$	M1	
		$y = 6x - 2\ln 2 + 2$		
		intersect: $3\mathrm{e}x + \mathrm{e} - 2 = 6x - 2\ln 2 + 2$		
		$x(3\mathrm{e} - 6) = 4 - \mathrm{e} - 2\ln 2$	M1	
		$x = \frac{4 - \mathrm{e} - 2\ln 2}{3\mathrm{e} - 6} = -0.0485$ (3sf)	A1	**(13)**
7.	*(a)*	$\arccos\theta = \frac{\pi}{3}, \quad \theta = \cos\frac{\pi}{3} = \frac{1}{2}$	M1 A1	
	(b)		B2 B3	
	(c)	let $\mathrm{f}(x) = \arccos(x - 1) - \sqrt{x+2}$		
		$\mathrm{f}(0) = 1.7,\ \mathrm{f}(1) = -0.16$	M1 A1	
		sign change, $\mathrm{f}(x)$ continuous $\therefore$ root	A1	
	(d)	$x_1 = 0.83944,\ x_2 = 0.88598,\ x_3 = 0.87233,$		
		$x_4 = 0.87632,\ x_5 = 0.87515,\ x_6 = 0.87549$	M1 A2	
		$\therefore\ \alpha = 0.875$ (3dp)	A1	**(14)**
			Total	**(75)**