

1. Given that

$$x = \sec^2 y + \tan y,$$

show that

$$\frac{dy}{dx} = \frac{\cos^2 y}{2 \tan y + 1}. \quad (4)$$

2. The functions f and g are defined by

$$f : x \rightarrow 3x - 4, \quad x \in \mathbb{R},$$

$$g : x \rightarrow \frac{2}{x+3}, \quad x \in \mathbb{R}, \quad x \neq -3.$$

(a) Evaluate $fg(1)$. (2)

(b) Solve the equation $gf(x) = 6$. (4)

3. Giving your answers to 2 decimal places, solve the simultaneous equations

$$e^{2y} - x + 2 = 0$$

$$\ln(x + 3) - 2y - 1 = 0 \quad (8)$$

4. (a) Use the derivatives of $\sin x$ and $\cos x$ to prove that

$$\frac{d}{dx}(\tan x) = \sec^2 x. \quad (4)$$

The tangent to the curve $y = 2x \tan x$ at the point where $x = \frac{\pi}{4}$ meets the y -axis at the point P .

(b) Find the y -coordinate of P in the form $k\pi^2$ where k is a rational constant. (6)

5. (a) Express $3 \cos x^\circ + \sin x^\circ$ in the form $R \cos(x - \alpha)^\circ$ where $R > 0$ and $0 < \alpha < 90$. (4)

(b) Using your answer to part (a), or otherwise, solve the equation

$$6 \cos^2 x^\circ + \sin 2x^\circ = 0,$$

for x in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place where appropriate. (6)

6.

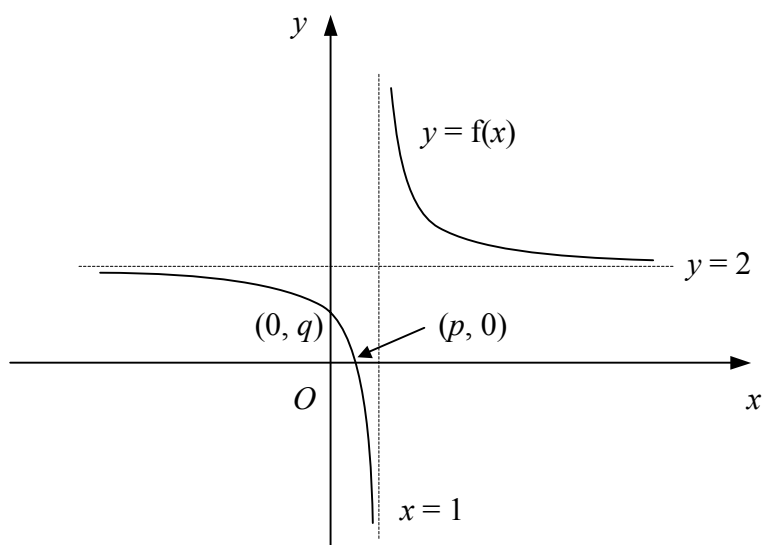


Figure 1

Figure 1 shows the curve with equation $y = f(x)$. The curve crosses the axes at $(p, 0)$ and $(0, q)$ and the lines $x = 1$ and $y = 2$ are asymptotes of the curve.

(a) Showing the coordinates of any points of intersection with the axes and the equations of any asymptotes, sketch on separate diagrams the graphs of

(i) $y = |f(x)|$,

(ii) $y = 2f(x + 1)$. (6)

Given also that

$$f(x) \equiv \frac{2x-1}{x-1}, \quad x \in \mathbb{R}, \quad x \neq 1,$$

(b) find the values of p and q , (3)

(c) find an expression for $f^{-1}(x)$. (3)

Turn over

7. (a) (i) Show that

$$\sin (x+30)^{\circ}+\sin (x-30)^{\circ} \equiv a \sin x^{\circ},$$

where a is a constant to be found.

- (ii) Hence find the exact value of $\sin 75^{\circ}+\sin 15^{\circ}$, giving your answer in the form $b\sqrt{6}$. (6)

- (b) Solve, for $0 \leq y \leq 360$, the equation

$$2 \cot ^2 y^{\circ}+5 \operatorname{cosec} y^{\circ}+\operatorname{cosec}^2 y^{\circ}=0 . \quad (6)$$

8.
$$f(x)=\frac{x^4+x^3-5 x^2-9}{x^2+x-6} .$$

- (a) Using algebraic division, show that

$$f(x)=x^2+A+\frac{B}{x+C},$$

where A, B and C are integers to be found. (5)

- (b) By sketching two suitable graphs on the same set of axes, show that the equation $f(x)=0$ has exactly one real root. (3)

- (c) Use the iterative formula

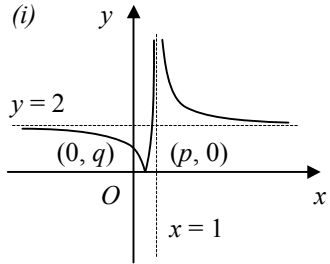
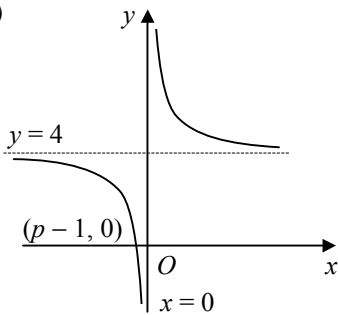
$$x_{n+1}=2+\frac{1}{x_n^2+1},$$

with a suitable starting value to find the root of the equation $f(x)=0$ correct to 3 significant figures and justify the accuracy of your answer. (5)

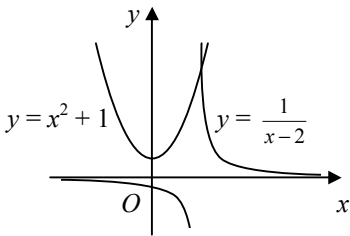
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C3 Paper A – Marking Guide

1.	$\frac{dx}{dy} = 2 \sec y \times \sec y \tan y + \sec^2 y = \sec^2 y (2 \tan y + 1) = \frac{2 \tan y + 1}{\cos^2 y}$	M1 A1	
	$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{\cos^2 y}{2 \tan y + 1}$	M1 A1	(4)
2.	(a) $= f\left(\frac{1}{2}\right) = -\frac{5}{2}$	M1 A1	
	(b) $gf(x) = \frac{2}{(3x-4)+3} = \frac{2}{3x-1}$	M1 A1	
	$\therefore \frac{2}{3x-1} = 6$		
	$2 = 6(3x-1)$	M1	
	$x = \frac{4}{9}$	A1	(6)
3.	$e^{2y} - x + 2 = 0 \Rightarrow e^{2y} = x - 2$		
	$2y = \ln(x-2)$	M1	
sub.	$\Rightarrow \ln(x+3) - \ln(x-2) - 1 = 0$	A1	
	$\ln \frac{x+3}{x-2} = 1$	M1	
	$\frac{x+3}{x-2} = e$	A1	
	$x+3 = e(x-2)$	M1	
	$3+2e = x(e-1)$	M1	
	$x = \frac{2e+3}{e-1} = 4.91 \text{ (2dp)}, y = \frac{1}{2} \ln\left(\frac{2e+3}{e-1} - 2\right) = 0.53 \text{ (2dp)}$	A2	(8)
4.	(a) $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$		
	$= \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x}$	M1 A1	
	$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$	M1 A1	
	(b) $\frac{dy}{dx} = 2 \times \tan x + 2x \times \sec^2 x = 2 \tan x + 2x \sec^2 x$	M1 A1	
	$x = \frac{\pi}{4}, y = \frac{\pi}{2}, \text{grad} = 2 + \pi$	B1	
	$\therefore y - \frac{\pi}{2} = (2 + \pi)(x - \frac{\pi}{4})$	M1	
	at P, $x = 0$		
	$\therefore y = \frac{\pi}{2} - \frac{\pi}{4}(2 + \pi) = -\frac{1}{4}\pi^2$	M1 A1	(10)
5.	(a) $3 \cos x + \sin x = R \cos x \cos \alpha + R \sin x \sin \alpha$		
	$R \cos \alpha = 3, R \sin \alpha = 1$		
	$\therefore R = \sqrt{3^2 + 1^2} = \sqrt{10}$	M1 A1	
	$\tan \alpha = \frac{1}{3}, \alpha = 18.4 \text{ (3sf)}$	M1 A1	
	$\therefore 3 \cos x^\circ + \sin x^\circ = \sqrt{10} \cos(x - 18.4)^\circ$		
	(b) $6 \cos^2 x + 2 \sin x \cos x = 0$	M1	
	$2 \cos x(3 \cos x + \sin x) = 0$	M1	
	$\cos x = 0$ or $3 \cos x + \sin x = \sqrt{10} \cos(x - 18.4) = 0$	A1	
	$x = 90, 270$ or $x - 18.4 = 90, 270$	B1 M1	
	$x = 90, 108.4 \text{ (1dp)}, 270, 288.4 \text{ (1dp)}$	A1	(10)

6. (a) (i)  (ii) 
- (b) $y = 0 \Rightarrow 2x - 1 = 0 \Rightarrow x = \frac{1}{2} \therefore p = \frac{1}{2}$ M1 A1
 $x = 0 \Rightarrow y = 1 \therefore q = 1$ B1
- (c) $y = \frac{2x-1}{x-1}, \quad y(x-1) = 2x-1$ M1
 $x(y-2) = y-1, \quad x = \frac{y-1}{y-2}$ M1
 $\therefore f^{-1}(x) = \frac{x-1}{x-2}$ A1 (12)

7. (a) (i) LHS = $\sin x \cos 30 + \cos x \sin 30 + \sin x \cos 30 - \cos x \sin 30$ M1 A1
 $= 2 \sin x \cos 30 = \sqrt{3} \sin x \quad [a = \sqrt{3}]$ A1
- (ii) let $x = 45, \sin 75 + \sin 15 = \sqrt{3} \sin 45 = \sqrt{3} \times \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{6}$ M2 A1
- (b) $2(\operatorname{cosec}^2 y - 1) + 5 \operatorname{cosec} y + \operatorname{cosec}^2 y = 0$ M1
 $3 \operatorname{cosec}^2 y + 5 \operatorname{cosec} y - 2 = 0, \quad (3 \operatorname{cosec} y - 1)(\operatorname{cosec} y + 2) = 0$ M1
 $\operatorname{cosec} y = -2 \text{ or } \frac{1}{3} \text{ (no solutions)}$ A1
 $\sin y = -\frac{1}{2}$
 $y = 180 + 30, 360 - 30$ B1 M1
 $y = 210, 330$ A1 (12)

8. (a)
$$\begin{array}{r} x^2 + 0x + 1 \\ x^2 + x - 6 \overline{) x^4 + x^3 - 5x^2 + 0x - 9} \\ \underline{x^4 + x^3 - 6x^2} \\ x^2 + 0x - 9 \\ \underline{x^2 + x - 6} \\ -x - 3 \end{array}$$
 M1 A1
- $\therefore f(x) = x^2 + 1 + \frac{-x-3}{x^2+x-6}$ A1
 $= x^2 + 1 - \frac{x+3}{(x-2)(x+3)} = x^2 + 1 - \frac{1}{x-2} \quad [A = 1, B = -1, C = -2]$ M1 A1
- (b)  B2
- $f(x) = 0 \Rightarrow x^2 + 1 = \frac{1}{x-2}, \text{ graphs intersect once } \therefore \text{ exactly one real root}$ B1
- (c) e.g. $x_0 = 3, x_1 = 2.1, x_2 = 2.1848, x_3 = 2.1732, x_4 = 2.1747$ M1 A1
 $\therefore \text{ root} = 2.17 \text{ (3sf)}$ A1
 $f(2.165) = -0.37, f(2.175) = 0.016$ M1
sign change, $f(x)$ continuous $\therefore$ root A1 (13)

Total (75)