1. Find the value of $y$ such that

$$4^{y+3} = 8.$$ **(3)**

---

2. Find

$$\int (3x^2 + \frac{1}{2x^2})\ dx.$$ **(4)**

---

3.

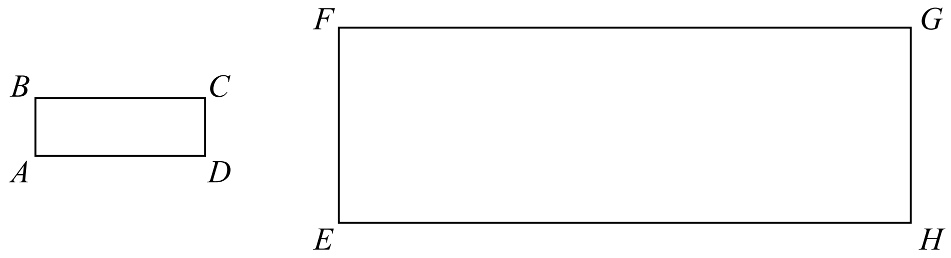


**Figure 1**

Figure 1 shows the rectangles $ABCD$ and $EFGH$ which are similar.

Given that $AB = (3 - \sqrt{5})$ cm, $AD = \sqrt{5}$ cm and $EF = (1 + \sqrt{5})$ cm, find the length $EH$ in cm, giving your answer in the form $a + b\sqrt{5}$ where $a$ and $b$ are integers. **(6)**

---

4. *(a)* Sketch on the same diagram the curves $y = x^2 - 4x$ and $y = -\frac{1}{x}$. **(4)**

*(b)* State, with a reason, the number of real solutions to the equation

$$x^2 - 4x + \frac{1}{x} = 0.$$ **(2)**

---

5. *(a)* By completing the square, find in terms of the constant $k$ the roots of the equation

$$x^2 + 2kx + 4 = 0.$$ **(4)**

*(b)* Hence find the exact roots of the equation

$$x^2 + 6x + 4 = 0.$$ **(2)**

---

6. *(a)* Evaluate

$$\sum_{r=1}^{50} (80 - 3r).$$ **(3)**

*(b)* Show that

$$\sum_{r=1}^{n} \frac{r+3}{2} = kn(n+7),$$

where $k$ is a rational constant to be found. **(4)**

---

7. Solve the simultaneous equations

$$x - 3y + 7 = 0$$

$$x^2 + 2xy - y^2 = 7$$ **(7)**

---

8. Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^3 - 4}{x^3}, \quad x \neq 0,$$

*(a)* find $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$. **(3)**

Given also that $y = 0$ when $x = -1$,

*(b)* find the value of $y$ when $x = 2$. **(6)**

---

***Turn over***

9. A curve has the equation $y = (\sqrt{x} - 3)^2, \ x \geq 0$.

*(a)* Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 - \dfrac{3}{\sqrt{x}}$. **(4)**

The point $P$ on the curve has $x$-coordinate 4.

*(b)* Find an equation for the normal to the curve at $P$ in the form $y = mx + c$. **(5)**

*(c)* Show that the normal to the curve at $P$ does not intersect the curve again. **(4)**

---

10. The straight line $l$ has gradient 3 and passes through the point $A$ $(-6, 4)$.

*(a)* Find an equation for $l$ in the form $y = mx + c$. **(2)**

The straight line $m$ has the equation $x - 7y + 14 = 0$.

Given that $m$ crosses the $y$-axis at the point $B$ and intersects $l$ at the point $C$,

*(b)* find the coordinates of $B$ and $C$, **(4)**

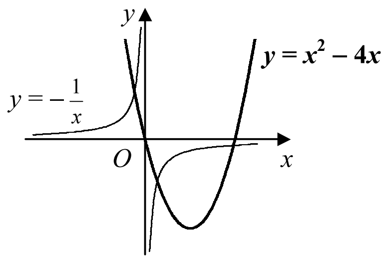
*(c)* show that $\angle BAC = 90°$, **(4)**

*(d)* find the area of triangle $ABC$. **(4)**

---

**END**

# C1 Paper K – Marking Guide

| | | | | |
|---|---|---|---|---|
| **1.** | $(2^2)^{y+3} = 2^3$ | | M1 | |
| | $2y + 6 = 3$ | | M1 | |
| | $y = -\frac{3}{2}$ | | A1 | **(3)** |
| **2.** | $= \int (3x^2 + \frac{1}{2}x^{-2})\ \mathrm{d}x$ | | B1 | |
| | $= x^3 - \frac{1}{2}x^{-1} + c$ | | M1 A2 | **(4)** |
| **3.** | $\frac{EH}{AD} = \frac{EF}{AB}\ \therefore\ \frac{EH}{\sqrt{5}} = \frac{1+\sqrt{5}}{3-\sqrt{5}}$ | | M1 | |
| | $\frac{1+\sqrt{5}}{3-\sqrt{5}} = \frac{1+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{3+\sqrt{5}+3\sqrt{5}+5}{9-5} = 2 + \sqrt{5}$ | | M2 A1 | |
| | $\therefore\ EH = \sqrt{5}\,(2 + \sqrt{5}\,) = 5 + 2\sqrt{5}$ | | M1 A1 | **(6)** |
| **4.** | *(a)* |  | B2<br>B2 | |
| | *(b)* | 3 solutions | B1 | |
| | | $x^2 - 4x + \frac{1}{x} = 0\ \Rightarrow\ x^2 - 4x = -\frac{1}{x}$ | | |
| | | and the graphs of $y = x^2 - 4x$ and $y = -\frac{1}{x}$ intersect at 3 points | B1 | **(6)** |
| **5.** | *(a)* | $(x + k)^2 - k^2 + 4 = 0$ | M1 | |
| | | $(x + k)^2 = k^2 - 4$ | A1 | |
| | | $x + k = \pm\sqrt{k^2 - 4}$ | M1 | |
| | | $x = -k \pm \sqrt{k^2 - 4}$ | A1 | |
| | *(b)* | $k = 3\quad \therefore\ x = -3 \pm \sqrt{3^2 - 4}$ | M1 | |
| | | $= -3 \pm \sqrt{5}$ | A1 | **(6)** |
| **6.** | *(a)* | AP: $a = 77,\ l = -70$ | B1 | |
| | | $S_{50} = \frac{50}{2}[77 + (-70)] = 25 \times 7 = 175$ | M1 A1 | |
| | *(b)* | AP: $a = 2,\ d = \frac{1}{2}$ | B2 | |
| | | $S_n = \frac{n}{2}[4 + \frac{1}{2}(n - 1)]$ | M1 | |
| | | $= \frac{1}{4}n[8 + (n - 1)] = \frac{1}{4}n(n + 7)\qquad [\,k = \frac{1}{4}\,]$ | A1 | **(7)** |
| **7.** | $x - 3y + 7 = 0\ \Rightarrow\ x = 3y - 7$ | | M1 | |
| | sub. into $x^2 + 2xy - y^2 = 7$ | | | |
| | $(3y - 7)^2 + 2y(3y - 7) - y^2 = 7$ | | M1 | |
| | $y^2 - 4y + 3 = 0$ | | A1 | |
| | $(y - 1)(y - 3) = 0$ | | M1 | |
| | $y = 1, 3$ | | A1 | |
| | $\therefore\ x = -4,\ y = 1$ or $x = 2,\ y = 3$ | | M1 A1 | **(7)** |

| | | | | |
|---|---|---|---|---|
| **8.** | *(a)* | $\frac{dy}{dx} = 1 - 4x^{-3}$ | B1 | |
| | | $\frac{d^2y}{dx^2} = 12x^{-4}$ | M1 A1 | |
| | *(b)* | $y = \int (1 - 4x^{-3})\ dx$ | | |
| | | $y = x + 2x^{-2} + c$ | M1 A2 | |
| | | $x = -1, y = 0 \ \therefore \ 0 = -1 + 2 + c$ | | |
| | | $c = -1$ | M1 | |
| | | $y = x + 2x^{-2} - 1$ | | |
| | | when $x = 2, \ y = 2 + \frac{1}{2} - 1 = \frac{3}{2}$ | M1 A1 | **(9)** |
| **9.** | *(a)* | $y = x - 6\sqrt{x} + 9$ | M1 A1 | |
| | | $\frac{dy}{dx} = 1 - 3x^{-\frac{1}{2}} = 1 - \frac{3}{\sqrt{x}}$ | M1 A1 | |
| | *(b)* | $x = 4 \ \therefore y = 1$ | | |
| | | grad of tangent $= 1 - \frac{3}{2} = -\frac{1}{2}$ | M1 | |
| | | grad of normal $= \frac{-1}{-\frac{1}{2}} = 2$ | M1 A1 | |
| | | $\therefore \ y - 1 = 2(x - 4)$ | M1 | |
| | | $y = 2x - 7$ | A1 | |
| | *(c)* | at intersect: $x - 6\sqrt{x} + 9 = 2x - 7$ | | |
| | | $x + 6\sqrt{x} - 16 = 0$ | M1 | |
| | | $(\sqrt{x} + 8)(\sqrt{x} - 2) = 0$ | M1 | |
| | | $\sqrt{x} = -8, 2$ | A1 | |
| | | $\sqrt{x} = 2 \Rightarrow x = 4$ (at $P$) | | |
| | | $\sqrt{x} = -8 \Rightarrow$ no real solutions $\therefore$ normal does not intersect again | A1 | **(13)** |
| **10.** | *(a)* | $y - 4 = 3(x + 6)$ | M1 | |
| | | $y = 3x + 22$ | A1 | |
| | *(b)* | at $B$, $x = 0 \ \therefore y = 2 \ \Rightarrow \ B\ (0, 2)$ | B1 | |
| | | at $C$, $x - 7(3x + 22) + 14 = 0$ | M1 | |
| | | $x = -7$ | A1 | |
| | | $\therefore \ C\ (-7, 1)$ | A1 | |
| | *(c)* | grad $AB = \frac{2-4}{0-(-6)} = -\frac{1}{3}$ | M1 A1 | |
| | | grad $AC = \frac{1-4}{-7-(-6)} = 3$ | | |
| | | grad $AB \times$ grad $AC = -\frac{1}{3} \times 3 = -1$ | M1 | |
| | | $\therefore$ $AB$ perp to $AC$ $\therefore$ $\angle BAC = 90°$ | A1 | |
| | *(d)* | $AB = \sqrt{(0+6)^2 + (2-4)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$ | M1 A1 | |
| | | $AC = \sqrt{(-7+6)^2 + (1-4)^2} = \sqrt{1+9} = \sqrt{10}$ | | |
| | | area $= \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10} = 10$ | M1 A1 | **(14)** |
| | | | Total | **(75)** |