

1. The points A , B and C have coordinates $(-3, 0)$, $(5, -2)$ and $(4, 1)$ respectively.

Find an equation for the straight line which passes through C and is parallel to AB .
Give your answer in the form $ax + by = c$, where a , b and c are integers. (4)

2. Express $\sqrt{22.5}$ in the form $k\sqrt{10}$. (4)
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3.

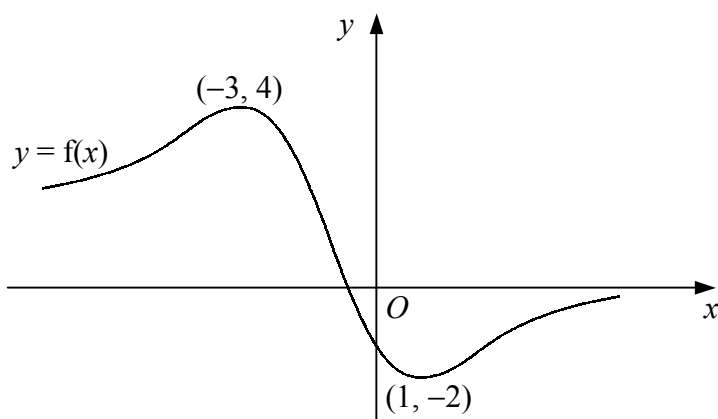


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve has a maximum at $(-3, 4)$ and a minimum at $(1, -2)$.

Showing the coordinates of any turning points, sketch on separate diagrams the curves with equations

(a) $y = 2f(x)$, (3)

(b) $y = -f(x)$. (3)

4. (a) Solve the inequality

$$4(x - 2) < 2x + 5. \quad (2)$$

- (b) Find the value of y such that

$$4^{y+1} = 8^{2y-1}. \quad (4)$$

5. A sequence of terms $\{t_n\}$ is defined for $n \geq 1$ by the recurrence relation

$$t_{n+1} = kt_n - 7, \quad t_1 = 3,$$

where k is a constant.

- (a) Find expressions for t_2 and t_3 in terms of k . (3)

Given that $t_3 = 13$,

- (b) find the possible values of k . (3)
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6. The curve with equation $y = \sqrt{8x}$ passes through the point A with x -coordinate 2.

Find an equation for the tangent to the curve at A . (7)

7. As part of a new training programme, Habib decides to do sit-ups every day.

He plans to do 20 per day in the first week, 22 per day in the second week, 24 per day in the third week and so on, increasing the daily number of sit-ups by two at the start of each week.

- (a) Find the number of sit-ups that Habib will do in the fifth week. (3)

- (b) Show that he will do a total of 1512 sit-ups during the first eight weeks. (2)

In the n th week of training, the number of sit-ups that Habib does is greater than 300 for the first time.

- (c) Find the value of n . (3)
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Turn over

8. Some ink is poured onto a piece of cloth forming a stain that then spreads.

The area of the stain, $A \text{ cm}^2$, after t seconds is given by

$$A = (p + qt)^2,$$

where p and q are positive constants.

Given that when $t = 0$, $A = 4$ and that when $t = 5$, $A = 9$,

(a) find the value of p and show that $q = \frac{1}{5}$, (5)

(b) find $\frac{dA}{dt}$ in terms of t , (4)

(c) find the rate at which the area of the stain is increasing when $t = 15$. (2)

9. The curve C has the equation $y = x^2 + 2x + 4$.

(a) Express $x^2 + 2x + 4$ in the form $a(x + b)^2 + c$ and hence state the coordinates of the minimum point of C . (4)

The straight line l has the equation $x + y = 8$.

(b) Sketch l and C on the same set of axes. (3)

(c) Find the coordinates of the points where l and C intersect. (4)

10. The curve C has the equation $y = f(x)$.

Given that

$$\frac{dy}{dx} = 3 - \frac{2}{x^2}, \quad x \neq 0,$$

and that the point A on C has coordinates $(2, 6)$,

(a) find an equation for C , (5)

(b) find an equation for the tangent to C at A , giving your answer in the form $ax + by + c = 0$ where a , b and c are integers, (4)

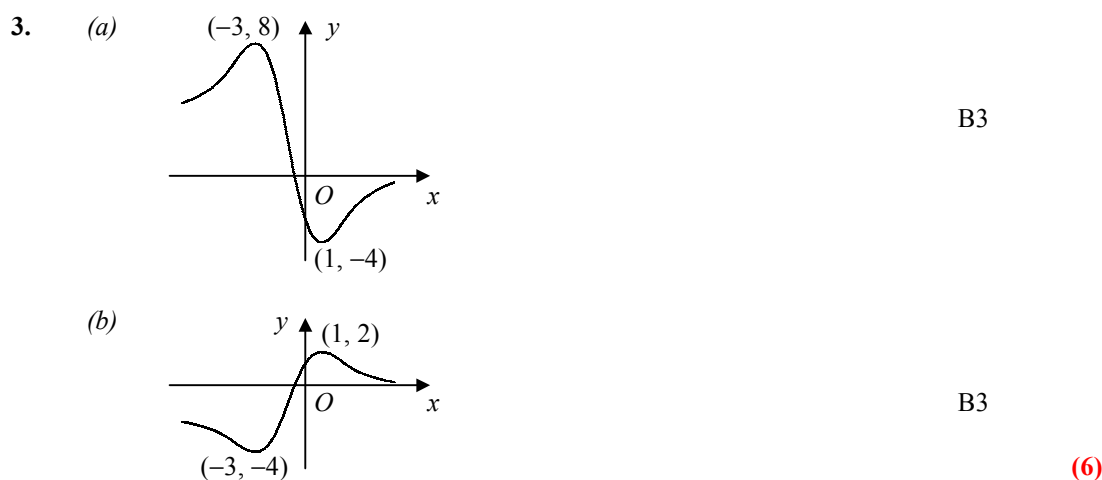
(c) show that the line $y = x + 3$ is also a tangent to C . (3)

END

C1 Paper J – Marking Guide

1. $\text{grad } AB = \frac{-2-0}{5-(-3)} = -\frac{1}{4}$ $\therefore y - 1 = -\frac{1}{4}(x - 4)$ $4y - 4 = -x + 4$ $x + 4y = 8$	M1 A1 M1 A1 (4)
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2. $= \sqrt{\frac{45}{2}} = \frac{3\sqrt{5}}{\sqrt{2}}$ $= \frac{3\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3}{2}\sqrt{10}$	M1 A1 M1 A1 (4)
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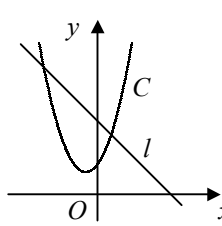
4. (a) $4x - 8 < 2x + 5$ $2x < 13$ $x < 6\frac{1}{2}$ (b) $(2^2)^{y+1} = (2^3)^{2y-1}$ $2^{2y+2} = 2^{6y-3}$ $2y + 2 = 6y - 3$ $y = \frac{5}{4}$	M1 A1 M1 A1 M1 A1 (6)
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5. (a) $t_2 = 3k - 7$ $t_3 = k(3k - 7) - 7 = 3k^2 - 7k - 7$ (b) $3k^2 - 7k - 7 = 13$ $3k^2 - 7k - 20 = 0$ $(3k + 5)(k - 4) = 0$ $k = -\frac{5}{3}, 4$	B1 M1 A1 M1 A2 (6)
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6. $x = 2 \therefore y = \sqrt{16} = 4$ $y = \sqrt{8}\sqrt{x} = 2\sqrt{2}x^{\frac{1}{2}}$ $\frac{dy}{dx} = \sqrt{2}x^{-\frac{1}{2}}$ $\text{grad} = \frac{\sqrt{2}}{\sqrt{2}} = 1$ $\therefore y - 4 = 1(x - 2) \quad [y = x + 2]$	B1 B1 M1 A1 M1 M1 A1 (7)
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7.	(a)	$a = 20 \times 7 = 140, d = 2 \times 7 = 14$ $u_5 = 140 + (4 \times 14) = 196$	B1 M1 A1
	(b)	$S_8 = \frac{8}{2} [280 + (7 \times 14)] = 4 \times 378 = 1512$	M1 A1
	(c)	$140 + 14(n-1) > 300$ $n > \frac{160}{14} + 1$ $n > 12\frac{3}{7} \therefore n = 13$	M1 M1 A1 (8)

8.	(a)	$t = 0, A = 4 \Rightarrow 4 = p^2$ $p > 0 \therefore p = 2$ $t = 5, A = 9 \Rightarrow 9 = (2 + 5q)^2$ $2 + 5q = \pm 3$ $q = \frac{1}{5}(-2 \pm 3)$ $q > 0 \therefore q = \frac{1}{5}$	M1 A1 M1 M1 A1
	(b)	$A = (2 + \frac{1}{5}t)^2 = 4 + \frac{4}{5}t + \frac{1}{25}t^2$ $\frac{dA}{dt} = \frac{4}{5} + \frac{2}{25}t$	M1 A1 M1 A1
	(c)	$t = 15 \therefore \frac{dA}{dt} = \frac{4}{5} + \frac{2}{25}(15) = 2 \text{ cm}^2 \text{ s}^{-1}$	M1 A1 (11)

9.	(a)	$x^2 + 2x + 4 = (x+1)^2 - 1 + 4$ $= (x+1)^2 + 3$ minimum: $(-1, 3)$	M1 A1 A2
	(b)		B2 B1
	(c)	$x^2 + 2x + 4 = 8 - x$ $x^2 + 3x - 4 = 0$ $(x+4)(x-1) = 0$ $x = -4, 1$ $\therefore (-4, 12) \text{ and } (1, 7)$	M1 A1 M1 A1 (11)

10.	(a)	$y = \int (3 - \frac{2}{x^2}) dx$ $y = 3x + 2x^{-1} + c$ $(2, 6) \therefore 6 = 6 + 1 + c$ $c = -1$ $y = 3x + 2x^{-1} - 1$	M1 A2 M1 A1
	(b)	grad $= 3 - \frac{1}{2} = \frac{5}{2}$ $y - 6 = \frac{5}{2}(x - 2)$ $2y - 12 = 5x - 10$ $5x - 2y + 2 = 0$	M1 A1 M1 A1
	(c)	$3x + 2x^{-1} - 1 = x + 3$ $3x^2 + 2 - x = x^2 + 3x$ $x^2 - 2x + 1 = 0$ $(x-1)^2 = 0, \text{ repeated root } \therefore \text{tangent}$	M1 M1 A1 (12)

Total (75)