

1. Evaluate

$$\sum_{r=1}^{30} (3r + 4). \quad (3)$$

2. (a) Express $x^2 + 6x + 7$ in the form $(x + a)^2 + b$. (3)

(b) State the coordinates of the minimum point of the curve $y = x^2 + 6x + 7$. (1)

3. The straight line l_1 has the equation $3x - y = 0$.
The straight line l_2 has the equation $x + 2y - 4 = 0$.

(a) Sketch l_1 and l_2 on the same diagram, showing the coordinates of any points where each line meets the coordinate axes. (3)

(b) Find, as exact fractions, the coordinates of the point where l_1 and l_2 intersect. (3)

4. Find the pairs of values (x, y) which satisfy the simultaneous equations

$$\begin{aligned} 3x^2 + y^2 &= 21 \\ 5x + y &= 7 \end{aligned} \quad (7)$$

5. (a) Sketch on the same diagram the graphs of $y = (x - 1)^2(x - 5)$ and $y = 8 - 2x$.

Label on your diagram the coordinates of any points where each graph meets the coordinate axes. (5)

(b) Explain how your diagram shows that there is only one solution, α , to the equation

$$(x - 1)^2(x - 5) = 8 - 2x. \quad (1)$$

(c) State the integer, n , such that

$$n < \alpha < n + 1. \quad (1)$$

6. The curve with equation $y = x^2 + 2x$ passes through the origin, O .

(a) Find an equation for the normal to the curve at O . (5)

(b) Find the coordinates of the point where the normal to the curve at O intersects the curve again. (3)

7. Given that

$$y = \sqrt{x} - \frac{4}{\sqrt{x}},$$

(a) find $\frac{dy}{dx}$, (3)

(b) find $\frac{d^2y}{dx^2}$, (2)

(c) show that

$$4x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} - y = 0. \quad (3)$$

8. (a) Prove that the sum of the first n positive integers is given by

$$\frac{1}{2}n(n+1). \quad (4)$$

(b) Hence, find the sum of

(i) the integers from 100 to 200 inclusive,

(ii) the integers between 300 to 600 inclusive which are divisible by 3. (5)

Turn over

9. (a) Express each of the following in the form $p + q\sqrt{2}$ where p and q are rational.

(i) $(4 - 3\sqrt{2})^2$

(ii) $\frac{1}{2 + \sqrt{2}}$ (5)

(b) (i) Solve the equation

$$y^2 + 8 = 9y.$$

(ii) Hence solve the equation

$$x^3 + 8 = 9x^{\frac{3}{2}}. \quad (5)$$

10.

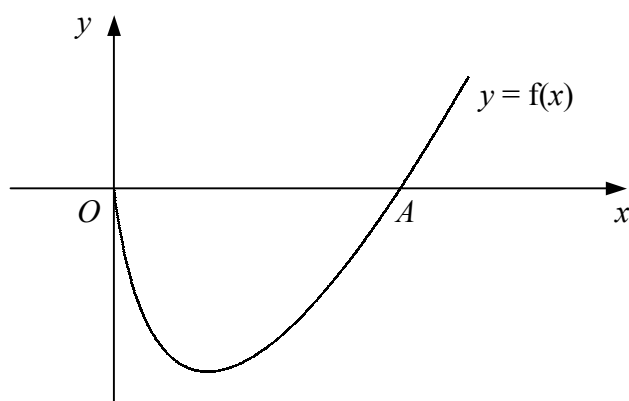


Figure 1

Figure 1 shows the curve with equation $y = f(x)$.

The curve meets the x -axis at the origin and at the point A .

Given that

$$f'(x) = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}},$$

(a) find $f(x)$, (5)

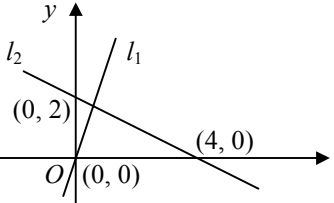
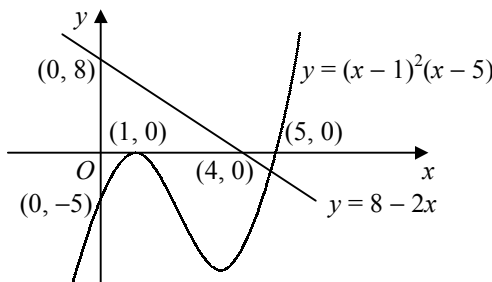
(b) find the coordinates of A . (2)

The point B on the curve has x -coordinate 2.

(c) Find an equation for the tangent to the curve at B in the form $y = mx + c$. (6)

END

C1 Paper H – Marking Guide

1.	AP: $a = 7, l = 94$ $S_{30} = \frac{30}{2}(7 + 94) = 15 \times 101 = 1515$	B1 M1 A1	(3)
2.	(a) $= (x + 3)^2 - 9 + 7$ $= (x + 3)^2 - 2$ (b) $(-3, -2)$	M1 A2 B1	(4)
3.	(a)  (b) $l_1 \Rightarrow 6x - 2y = 0$ $l_2: x + 2y - 4 = 0$ adding $7x - 4 = 0, x = \frac{4}{7}$ $\therefore$ intersect at $(\frac{4}{7}, \frac{12}{7})$	B2 B1 M1 A1 A1	(6)
4.	$5x + y = 7 \Rightarrow y = 7 - 5x$ sub. into $3x^2 + y^2 = 21$ $3x^2 + (7 - 5x)^2 = 21$ $2x^2 - 5x + 2 = 0$ $(2x - 1)(x - 2) = 0$ $x = \frac{1}{2}, 2$ $\therefore (\frac{1}{2}, \frac{9}{2})$ and $(2, -3)$	M1 M1 A1 M1 A1 M1 A1	(7)
5.	(a)  (b) the graphs intersect at exactly one point $\therefore$ one solution (c) $n = 4$	B3 B2 B1 B1	(7)
6.	(a) $\frac{dy}{dx} = 2x + 2$ grad of tangent = 2 grad of normal = $-\frac{1}{2} = -\frac{1}{2}$ $\therefore y = -\frac{1}{2}x$ (b) $x^2 + 2x = -\frac{1}{2}x$ $2x^2 + 5x = 0, x(2x + 5) = 0$ $x = 0$ (at O), $-\frac{5}{2}$ $\therefore (-\frac{5}{2}, \frac{5}{4})$	M1 A1 A1 M1 A1 M1 A1 A1	(8)

7.	(a)	$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$	M1 A2	
	(b)	$\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-\frac{3}{2}} - 3x^{-\frac{5}{2}}$	M1 A1	
	(c)	$\begin{aligned} \text{LHS} &= 4x^2(-\frac{1}{4}x^{-\frac{3}{2}} - 3x^{-\frac{5}{2}}) + 4x(\frac{1}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}) - (x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}) \\ &= -x^{\frac{1}{2}} - 12x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} + 8x^{-\frac{1}{2}} - x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} \\ &= 0 \end{aligned}$	M1 A1 A1	(8)
8.	(a)	$\begin{aligned} S_n &= 1 + 2 + 3 + \dots + n \\ S_n &= n + (n-1) + (n-2) + \dots + 1 \\ \text{adding, } 2S_n &= n(n+1) \\ S_n &= \frac{1}{2}n(n+1) \end{aligned}$	B1 M1 M1 A1	
	(b)	$\begin{aligned} (i) \quad &= S_{200} - S_{99} \\ &= \frac{1}{2} \times 200 \times 201 - \frac{1}{2} \times 99 \times 100 \\ &= 20\,100 - 4950 = 15\,150 \\ (ii) \quad &= 3 \times 15\,150 = 45\,450 \end{aligned}$	M1 M1 A1 M1 A1	(9)
9.	(a)	$\begin{aligned} (i) \quad &= 16 - 24\sqrt{2} + 18 = 34 - 24\sqrt{2} \\ (ii) \quad &= \frac{1}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} \\ &= \frac{2-\sqrt{2}}{4-2} = 1 - \frac{1}{2}\sqrt{2} \end{aligned}$	M1 A1 M1 M1 A1	
	(b)	$\begin{aligned} (i) \quad &y^2 - 9y + 8 = 0 \\ &(y-1)(y-8) = 0 \\ &y = 1, 8 \\ (ii) \quad &\text{let } y = x^{\frac{3}{2}} \Rightarrow y^2 + 8 = 9y \\ &\therefore x^{\frac{3}{2}} = 1, 8 \\ &x = 1 \text{ or } (\sqrt[3]{8})^2 \\ &x = 1 \text{ or } 4 \end{aligned}$	M1 A1 B1 M1 A1	(10)
10.	(a)	$\begin{aligned} f(x) &= \int (3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}) dx \\ f(x) &= 2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} + c \\ (0, 0) \therefore c &= 0 \\ f(x) &= 2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} \end{aligned}$	M1 A2 M1 A1	
	(b)	$\begin{aligned} 2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} &= 0 \\ 2x^{\frac{1}{2}}(x-4) &= 0 \\ x &= 0 \text{ (at } O), 4 \therefore A(4, 0) \end{aligned}$	M1 A1	
	(c)	$\begin{aligned} x &= 2 \therefore y = 2(2\sqrt{2}) - 8(\sqrt{2}) = -4\sqrt{2} \\ \text{grad} &= 3\sqrt{2} - \frac{4}{\sqrt{2}} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2} \\ \therefore y + 4\sqrt{2} &= \sqrt{2}(x-2) \\ y &= \sqrt{2}x - 6\sqrt{2} \end{aligned}$	M1 A1 M1 A1 M1 A1	(13)
				Total (75)