1. Find in exact form the real solutions of the equation

$$x^4 = 5x^2 + 14.$$ **(3)**

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2. Express

$$\frac{2}{3\sqrt{5}+7}$$

in the form $a + b\sqrt{5}$ where a and b are rational. **(3)**

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3. *(a)* Solve the equation

$$x^{\frac{3}{2}} = 27.$$ **(2)**

(b) Express $(2\tfrac{1}{4})^{-\frac{1}{2}}$ as an exact fraction in its simplest form. **(2)**

4.

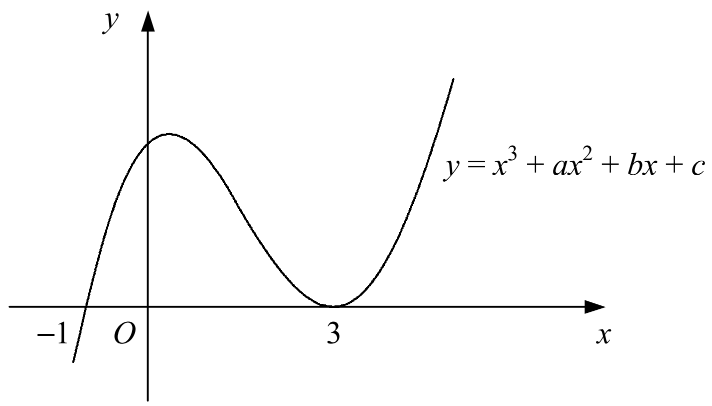

Figure 1

Figure 1 shows the curve with equation $y = x^3 + ax^2 + bx + c$, where a, b and c are constants. The curve crosses the x-axis at the point $(-1, 0)$ and touches the x-axis at the point $(3, 0)$.

Show that $a = -5$ and find the values of b and c. **(5)**

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5. Given that

$$y = \frac{x^4 - 3}{2x^2},$$

(a) find $\dfrac{dy}{dx}$, **(4)**

(b) show that $\dfrac{d^2y}{dx^2} = \dfrac{x^4 - 9}{x^4}$. **(2)**

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6. *(a)* Sketch on the same diagram the curve with equation $y = (x - 2)^2$ and the straight line with equation $y = 2x - 1$.

Label on your sketch the coordinates of any points where each graph meets the coordinate axes. **(5)**

(b) Find the set of values of x for which

$$(x - 2)^2 > 2x - 1.$$ **(3)**

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7. A curve has the equation $y = \dfrac{x}{2} + 3 - \dfrac{1}{x}$, $x \neq 0$.

The point A on the curve has x-coordinate 2.

(a) Find the gradient of the curve at A. **(4)**

(b) Show that the tangent to the curve at A has equation

$$3x - 4y + 8 = 0.$$ **(3)**

The tangent to the curve at the point B is parallel to the tangent at A.

(c) Find the coordinates of B. **(3)**

Turn over

8. The straight line l_1 has gradient $\frac{3}{2}$ and passes through the point A (5, 3).

(a) Find an equation for l_1 in the form $y = mx + c$. **(2)**

The straight line l_2 has the equation $3x - 4y + 3 = 0$ and intersects l_1 at the point B.

(b) Find the coordinates of B. **(3)**

(c) Find the coordinates of the mid-point of AB. **(2)**

(d) Show that the straight line parallel to l_2 which passes through the mid-point of AB also passes through the origin. **(4)**

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9. The third term of an arithmetic series is $5\frac{1}{2}$.

The sum of the first four terms of the series is $22\frac{3}{4}$.

(a) Show that the first term of the series is $6\frac{1}{4}$ and find the common difference. **(7)**

(b) Find the number of positive terms in the series. **(3)**

(c) Hence, find the greatest value of the sum of the first n terms of the series. **(2)**

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10. The curve C has the equation $y = \mathrm{f}(x)$.

Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 8x - \frac{2}{x^3}, \quad x \neq 0,$$

and that the point P (1, 1) lies on C,

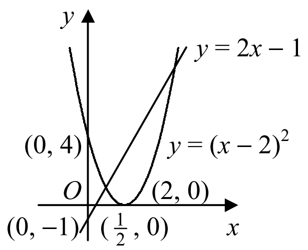
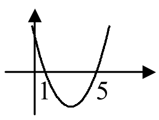
(a) find an equation for the tangent to C at P in the form $y = mx + c$, **(3)**

(b) find an equation for C, **(5)**

(c) find the x-coordinates of the points where C meets the x-axis, giving your answers in the form $k\sqrt{2}$. **(5)**

END

C1 Paper F – Marking Guide

1.		$x^4 - 5x^2 - 14 = 0, \quad (x^2 + 2)(x^2 - 7) = 0$	M1	
		$x^2 = -2$ (no solutions) or 7	A1	
		$x = \pm\sqrt{7}$	A1	**(3)**
2.		$= \frac{2}{3\sqrt{5}+7} \times \frac{3\sqrt{5}-7}{3\sqrt{5}-7} = \frac{6\sqrt{5}-14}{45-49} = \frac{7}{2} - \frac{3}{2}\sqrt{5}$	M2 A1	**(3)**
3.	*(a)*	$x = (\sqrt[3]{27})^2 = 3^2 = 9$	M1 A1	
	(b)	$= (\frac{9}{4})^{-\frac{1}{2}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$	M1 A1	**(4)**
4.		cubic, coeff of $x^3 = 1$, crosses x-axis at $(-1, 0)$, touches at $(3, 0)$		
		$\therefore\ y = (x + 1)(x - 3)^2$	M1 A1	
		$= (x + 1)(x^2 - 6x + 9)$		
		$= x^3 - 6x^2 + 9x + x^2 - 6x + 9$	M1	
		$= x^3 - 5x^2 + 3x + 9$		
		$\therefore\ a = -5,\ b = 3,\ c = 9$	A2	**(5)**
5.	*(a)*	$y = \frac{1}{2}x^2 - \frac{3}{2}x^{-2}$	M1 A1	
		$\frac{dy}{dx} = x + 3x^{-3}$	M1 A1	
	(b)	$\frac{d^2y}{dx^2} = 1 - 9x^{-4} = \frac{x^4 - 9}{x^4}$	M1 A1	**(6)**
6.	*(a)*		B2 B3	
	(b)	$x^2 - 4x + 4 > 2x - 1$		
		$x^2 - 6x + 5 > 0$		
		$(x - 1)(x - 5) > 0$ 	M1	
			M1	
		$x < 1$ or $x > 5$	A1	**(8)**
7.	*(a)*	$\frac{dy}{dx} = \frac{1}{2} + x^{-2}$	M1 A1	
		grad $= \frac{1}{2} + 2^{-2} = \frac{3}{4}$	M1 A1	
	(b)	$x = 2 \quad \therefore\ y = \frac{7}{2}$	B1	
		$y - \frac{7}{2} = \frac{3}{4}(x - 2)$	M1	
		$4y - 14 = 3x - 6$		
		$3x - 4y + 8 = 0$	A1	
	(c)	at B, grad $= \frac{3}{4}$		
		$\therefore\ \frac{1}{2} + x^{-2} = \frac{3}{4}$	M1	
		$x^2 = 4, \quad x = 2$ (at A), -2	A1	
		$\therefore\ B\,(-2, \frac{5}{2})$	A1	**(10)**

8.	*(a)*	$y - 3 = \frac{3}{2}(x - 5)$	M1	
		$y = \frac{3}{2}x - \frac{9}{2}$	A1	
	(b)	$3x - 4(\frac{3}{2}x - \frac{9}{2}) + 3 = 0$	M1	
		$x = 7$	A1	
		$\therefore B\ (7, 6)$	A1	
	(c)	$= (\frac{5+7}{2}, \frac{3+6}{2}) = (6, \frac{9}{2})$	M1 A1	
	(d)	l_2: $y = \frac{3}{4}x + \frac{3}{4}$ $\therefore$ grad $= \frac{3}{4}$	B1	
		$\therefore\ y - \frac{9}{2} = \frac{3}{4}(x - 6)$	M1	
		$y = \frac{3}{4}x$	A1	
		when $x = 0, y = 0$ $\therefore$ passes through origin	A1	**(11)**
9.	*(a)*	$a + 2d = 5\frac{1}{2}$ (1)	B1	
		$\frac{4}{2}(2a + 3d) = 22\frac{3}{4}$ (2)	M1 A1	
		(2) $\Rightarrow$ $4a + 6d = 22\frac{3}{4}$		
		(1) $\Rightarrow$ $3a + 6d = 16\frac{1}{2}$		
		subtracting, $a = 22\frac{3}{4} - 16\frac{1}{2} = 6\frac{1}{4}$	M1 A1	
		$d = \frac{1}{2}(5\frac{1}{2} - 6\frac{1}{4}) = -\frac{3}{8}$	M1 A1	
	(b)	$6\frac{1}{4} - \frac{3}{8}(n - 1) > 0$	M1	
		$50 - 3(n - 1) > 0$		
		$n < 17\frac{2}{3}$ $\therefore$ 17 positive terms	M1 A1	
	(c)	$= S_{17} = \frac{17}{2}[12\frac{1}{2} + (16 \times -\frac{3}{8})]$	M1	
		$= \frac{17}{2}(12\frac{1}{2} - 6) = \frac{17}{2} \times \frac{13}{2} = \frac{221}{4} = 55\frac{1}{4}$	A1	**(12)**
10.	*(a)*	grad $= 8 - 2 = 6$	B1	
		$\therefore\ y - 1 = 6(x - 1)$	M1	
		$y = 6x - 5$	A1	
	(b)	$y = \int (8x - \frac{2}{x^3})\ dx$		
		$y = 4x^2 + x^{-2} + c$	M1 A2	
		$(1, 1)$ $\therefore$ $1 = 4 + 1 + c$		
		$c = -4$	M1	
		$y = 4x^2 + x^{-2} - 4$	A1	
	(c)	$4x^2 + x^{-2} - 4 = 0$		
		$4x^4 - 4x^2 + 1 = 0$	M1	
		$(2x^2 - 1)^2 = 0$	M1	
		$x^2 = \frac{1}{2}$		
		$x = \pm\frac{1}{\sqrt{2}}$	A1	
		$x = \pm\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{1}{2}\sqrt{2}$	M1 A1	**(13)**

Total **(75)**