

1.
$$f(x) = (\sqrt{x} + 3)^2 + (1 - 3\sqrt{x})^2.$$

Show that $f(x)$ can be written in the form $ax + b$ where a and b are integers to be found. (3)

2. The curve C has the equation

$$y = x^2 + ax + b,$$

where a and b are constants.

Given that the minimum point of C has coordinates $(-2, 5)$, find the values of a and b . (4)

3. The sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_n = 2^n + kn,$$

where k is a constant.

Given that $u_1 = u_3$,

(a) find the value of k , (3)

(b) find the value of u_5 . (2)

4. Given that

$$\frac{dy}{dx} = 2x^3 + 1,$$

and that $y = 3$ when $x = 0$, find the value of y when $x = 2$. (6)

5.
$$f(x) = 4x - 3x^2 - x^3.$$

(a) Fully factorise $4x - 3x^2 - x^3$. (3)

(b) Sketch the curve $y = f(x)$, showing the coordinates of any points of intersection with the coordinate axes. (3)

6. The straight line l has the equation $x - 2y = 12$ and meets the coordinate axes at the points A and B .

Find the distance of the mid-point of AB from the origin, giving your answer in the form $k\sqrt{5}$.

(6)

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7. (a) Given that $y = 2^x$, find expressions in terms of y for

(i) 2^{x+2} ,

(ii) 2^{3-x} .

(4)

- (b) Show that using the substitution $y = 2^x$, the equation

$$2^{x+2} + 2^{3-x} = 33$$

can be rewritten as

$$4y^2 - 33y + 8 = 0.$$

(2)

- (c) Hence solve the equation

$$2^{x+2} + 2^{3-x} = 33.$$

(4)

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8. Given that

$$y = 2x^{\frac{3}{2}} - 1,$$

- (a) find $\frac{d^2y}{dx^2}$,

(3)

- (b) show that

$$4x^2 \frac{d^2y}{dx^2} - 3y = k,$$

where k is an integer to be found,

(2)

- (c) find

$$\int y^2 \, dx.$$

(6)

Turn over

9. The second and fifth terms of an arithmetic series are 26 and 41 respectively.

(a) Show that the common difference of the series is 5. (4)

(b) Find the 12th term of the series. (3)

Another arithmetic series has first term -12 and common difference 7.

Given that the sums of the first n terms of these two series are equal,

(c) find the value of n . (4)

10.

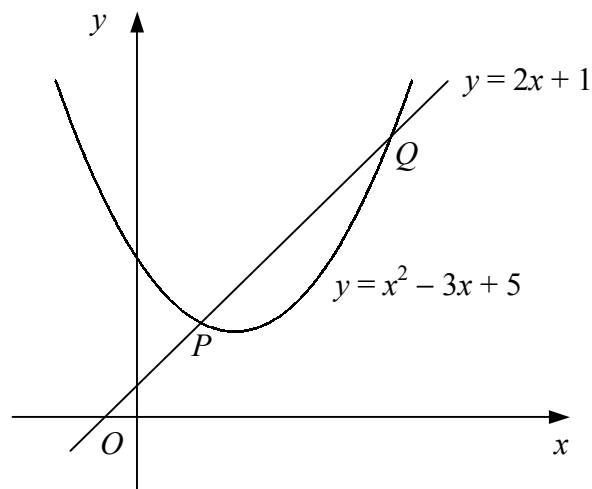


Figure 1

Figure 1 shows the curve $y = x^2 - 3x + 5$ and the straight line $y = 2x + 1$. The curve and line intersect at the points P and Q.

(a) Using algebra, show that P has coordinates (1, 3) and find the coordinates of Q. (4)

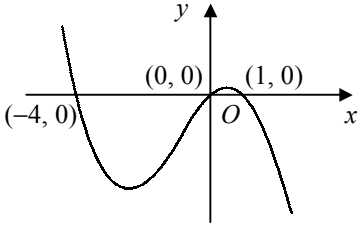
(b) Find an equation for the tangent to the curve at P. (4)

(c) Show that the tangent to the curve at Q has the equation $y = 5x - 11$. (2)

(d) Find the coordinates of the point where the tangent to the curve at P intersects the tangent to the curve at Q. (3)

END

C1 Paper B – Marking Guide

1.	$f(x) = x + 6\sqrt{x} + 9 + 1 - 6\sqrt{x} + 9x$ $= 10x + 10, \quad a = 10, b = 10$	M1 A1 A1	(3)
2.	quadratic, coeff of $x^2 = 1$, minimum $(-2, 5)$ $\therefore y = (x+2)^2 + 5$ $= x^2 + 4x + 9, \quad a = 4, b = 9$	M1 A1 M1 A1	(4)
3.	(a) $u_1 = 2 + k$ $u_3 = 8 + 3k$ $u_1 = u_3 \therefore 2 + k = 8 + 3k$ $k = -3$ (b) $u_5 = 2^5 - 3(5) = 32 - 15 = 17$	B1 M1 A1 M1 A1	(5)
4.	$y = \int (2x^3 + 1) \, dx$ $y = \frac{1}{2}x^4 + x + c$ $x = 0, y = 3 \therefore c = 3$ $y = \frac{1}{2}x^4 + x + 3$ when $x = 2, y = 8 + 2 + 3 = 13$	M1 A2 B1 M1 A1	(6)
5.	(a) $= x(4 - 3x - x^2)$ $= x(1 - x)(4 + x)$ (b) 	M1 M1 A1 B3	(6)
6.	$x = 0 \Rightarrow y = -6 \therefore (0, -6)$ $y = 0 \Rightarrow x = 12 \therefore (12, 0)$ mid-point $= (\frac{0+12}{2}, \frac{-6+0}{2}) = (6, -3)$ dist. from $O = \sqrt{6^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45}$ $= \sqrt{9 \times 5} = 3\sqrt{5}$	B1 M1 A1 M1 M1 A1	(6)
7.	(a) (i) $2^{x+2} = 2^2 \times 2^x = 4y$ (ii) $2^{3-x} = \frac{2^3}{2^x} = \frac{8}{y}$ (b) $2^{x+2} + 2^{3-x} = 33 \Rightarrow 4y + \frac{8}{y} = 33$ $4y^2 + 8 = 33y$ $4y^2 - 33y + 8 = 0$ (c) $(4y - 1)(y - 8) = 0$ $y = \frac{1}{4}, 8$ $2^x = \frac{1}{4}, 8$ $x = -2, 3$	M1 A1 M1 A1 M1 A1 M1 A1 A2	(10)

8.	(a)	$\frac{dy}{dx} = 3x^{\frac{1}{2}}$	M1 A1	
		$\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{1}{2}}$	A1	
	(b)	$\begin{aligned} \text{LHS} &= 4x^2\left(\frac{3}{2}x^{-\frac{1}{2}}\right) - 3(2x^{\frac{3}{2}} - 1) \\ &= 6x^{\frac{3}{2}} - 6x^{\frac{3}{2}} + 3 \\ &= 3 \quad [k = 3] \end{aligned}$	M1 A1	
	(c)	$\begin{aligned} &= \int (2x^{\frac{3}{2}} - 1)^2 dx \\ &= \int (4x^3 - 4x^{\frac{3}{2}} + 1) dx \\ &= x^4 - \frac{8}{5}x^{\frac{5}{2}} + x + c \end{aligned}$	M1 A1 M1 A3	(11)
9.	(a)	$a + d = 26$ $a + 4d = 41$ subtracting, $3d = 15$ $d = 5$	M1 A1 M1 A1	
	(b)	$a = 21$ $u_{12} = 21 + (11 \times 5) = 76$	B1 M1 A1	
	(c)	$\frac{n}{2}[42 + 5(n - 1)] = \frac{n}{2}[-24 + 7(n - 1)]$ $n(5n + 37) = n(7n - 31)$ $2n(n - 34) = 0$ $n > 0 \therefore n = 34$	M1 A1 M1 A1	(11)
10.	(a)	$x^2 - 3x + 5 = 2x + 1$ $x^2 - 5x + 4 = 0$ $(x - 1)(x - 4) = 0$ $x = 1, 4$ when $x = 1$, $y = 2(1) + 1 = 3$ $\therefore P(1, 3), Q(4, 9)$	M1 M1 A1 A1	
	(b)	$\frac{dy}{dx} = 2x - 3$ grad = -1 $\therefore y - 3 = -(x - 1) \quad [y = 4 - x]$	M1 A1 M1 A1	
	(c)	grad = 5 $\therefore y - 9 = 5(x - 4)$ $y - 9 = 5x - 20$ $y = 5x - 11$	M1 A1	
	(d)	$4 - x = 5x - 11$ $x = \frac{5}{2}$ $\therefore \left(\frac{5}{2}, \frac{3}{2}\right)$	M1 A1 A1	(13)
Total				(75)

Performance Record – C1 Paper B

[illegible]